An extent-based GQT-style unified implicature account of bare and modified numerals

3 · more/less than 3 · at most/least 3

Teodora Mihoc (Harvard University)

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Outline

The classic GQT theory of BNs, CMs, and SMs

An extent based GQT-style unified implicature account of BNs, CMs, and SMs

Additional results

Conclusion

Generalized Quantifier Theory

[Barwise and Cooper, 1981]

(1) $[\![\text{every}]\!] = \lambda P \,.\, \lambda Q \,.\, P \subseteq Q$

(2) $[\![\text{no}]\!] = \lambda P \,.\, \lambda Q \,.\, P \cap Q = \varnothing$

(3) $[\![\text{a}]\!] = \lambda P \,.\, \lambda Q \,.\, P \cap Q \neq \varnothing$

(4) $[\![\text{three}]\!] = \lambda P \,.\, \lambda Q \,.\, |P \cap Q| \geq 3$

(5) $[\![\text{more than three}]\!] = \lambda P \,.\, \lambda Q \,.\, |P \cap Q| > 3$

(6) $[\![\text{less than three}]\!] = \lambda P \,.\, \lambda Q \,.\, |P \cap Q| < 3$

(7) $[\![\text{at least three}]\!] = \lambda P \,.\, \lambda Q \,.\, |P \cap Q| \geq 3$

(8) $[\![\text{at most three}]\!] = \lambda P \,.\, \lambda Q \,.\, |P \cap Q| \leq 3$

(9) $[\![\text{exactly three}]\!] = \lambda P \,.\, \lambda Q \,.\, |P \cap Q| = 3$

(10) $[\![\text{between three and five}\ ]\!] = \lambda P \,.\, \lambda Q \,.\, 3 \leq |P \cap Q| \leq 5$

Features and bugs

- ⋆ Uniformity of DPs
- ⋆ Uniformity of natural language determiners

⋆ Uniformity of bare (BNs, *three*), comparative-modified (CMs, *more/less than three*), and superlative-modified numerals (SMs, *at least/most three*)

Challenged with data pointing to non-uniformity!
Challenges led to theories very different from GQT.
Where exactly does GQT fail?
We will assess it w.r.t. four major yardsticks:

entailments	scalar implicatures	ignorance	accept in DE env

✓ Entailments

[Horn, 1972, van Benthem, 1986, Krifka, 1999, Geurts and Nouwen, 2007, Buccola and Spector, 2016]

(11) a. Alice has 3 / more than 3 / at least 3 diamonds.
b. ¬ The number of diamonds that Alice has is 2 or less / 3 or less / 2 or less.
c. Alice has 3 / more than 3 / at least 3 diamonds, # if not less.

(12) a. Alice has less than 3 / at most 3 diamonds.
b. ¬ The number of diamonds that Alice has is 3 or more / 4 or more.
c. Alice has less than 3 / at most 3 diamonds, # if not more.

The upper bound of BNs as a scalar implicature

[Horn, 1972, Spector, 2013]

(13) a. Alice has 3 diamonds.
  b. ¬ The number of diamonds that Alice has is 4 or more.
  c. Alice has 3 diamonds, if not more.

⋆ *3 P Q* ambiguous between 'at least 3 P Q' and 'exactly 3 P Q'

⋆ One way to get this is to say that *3 P Q* entails 'at least 3 P Q', derives 'not at least 4 P Q' via scalar implicature.

⋆ Predicted scalar alternatives of BNs, CMs, and SMs:

(14) a. ScalAlts(*3 P Q*)
  = {…, *2 P Q*, *4 P Q*, …}
  b. ScalAlts(*more/less than 3 P Q*)
  = {…, *more/less than 2 P Q*, *more/less than 4 P Q*, …}
  c. ScalAlts(*at most/least 3 P Q*)
  = {…, *at most/least 2 P Q*, *at most/least 4 P Q*, …}

✓/✗ Scalar implicatures

[Krifka, 1999, Fox and Hackl, 2006, Mayr, 2013, Coppock and Brochhagen, 2013, Kennedy, 2015, Spector, 2015]

- Unembedded:

(15) Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
$\rightsquigarrow$ ¬ Alice has 4 / *more than 4 / *less than 2 / *at most 2 / *at least 4 diamonds.
(Total predicted meaning: She has exactly 3 / exactly 4 / exactly 2 / exactly 3 / exactly 3 diamonds.)

- In the scope of a universal operator:

(16) Alice is required to have 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
$\rightsquigarrow$ ¬ Alice is required to have 4 / more than 4 / less than 2 / at most 2 / at least 4 diamonds.

✓/✗ Scalar implicatures

[Mayr, 2013]

- ⋆ In the antecedent of a conditional:

(17) If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds she wins.
$\rightsquigarrow$ $\neg$ If Alice has 2 / more than 2 / less than 4 / at most 4 / at least 2 diamonds she wins.

- ⋆ In the scope of negation:

(18) Alice doesn't have 3 / more than 3 / less than 3 / *at most 3 / *at least 3 diamonds.
$\rightsquigarrow$ $\neg$ Alice doesn't have *2 / *more than 2 / *less than 4 / *at most 4 / *at least 2 diamonds.
(Total predicted meaning: She has exactly 2 / exactly 3 / exactly 3 / exactly 4 / exactly 2 diamonds.)

✓ Scalar implicatures

[Cummins et al., 2012]

⋆ Unembedded, *coarse granularity scale*:

(19) (example from [Spector, 2014, 42])

Context: Grades are attributed on the basis of the number of problems solved. People who solve between 1 and 5 problems get a C. People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.

John solved more than 5 problems. Peter solved more than 9.
$\rightsquigarrow$ $\neg$ John solved more than 9.

✗ Ignorance

[Geurts and Nouwen, 2007, Nouwen, 2010, Nouwen, 2015, Coppock and Brochhagen, 2013, Kennedy, 2015, Mendia, 2015, Spector, 2015]

- ⋆ Unembedded:

(20) Alice has 3 diamonds.
(*$\rightsquigarrow$ The speaker is not sure whether Alice has 3 or 4 or . . . .)

(21) Alice has more than 3 / less than 3 diamonds.
($\rightsquigarrow$ The speaker is not sure whether Alice has 4 or 5 or . . . / 2 or 1 or . . .)

(22) Alice has at least 3 / at most 3 diamonds.
*($\rightsquigarrow$ The speaker is not sure whether Alice has 3 or 4 or . . . / 3 or 2 or . . .)

✗ Ignorance

[Kennedy, 2015]

⋆ In the scope of a universal operator:

(23) Alice is required to have 3 diamonds.
$\not\rightsquigarrow$ The speaker is not sure whether Alice is required to have 3 or 4 or . . .

(24) Alice is required to have more than 3 / less than 3 / at most 3 / at least 3 diamonds.
($\rightsquigarrow$ The speaker is not sure whether Alice is required to have 4 or 5 or . . . / 2 or 1 or . . . / 3 or 2 or . . . / 3 or 4 or . . . .)

✗ Ignorance

⋆ In the scope of negation:

(25) Alice doesn't have 3 diamonds.
$\not\rightsquigarrow$ The speaker is not sure whether Alice doesn't have 3 or 4 or . . .

(26) Alice doesn't have more than 3 / less than 3 diamonds.
($\rightsquigarrow$ The speaker is not sure whether Alice has 3 or 2 or . . . / 3 or 4 or . . .)

✗ Acceptability in DE environments

[Nilsen, 2007, Geurts and Nouwen, 2007, Cohen and Krifka, 2014, Spector, 2015]

⋆ In the scope of negation:

(27) Alice doesn't have 3 / more than 3 / less than 3 diamonds.
→ Alice has 2 or less / 3 or less / 3 or more diamonds. ✓

(28) Alice doesn't have *at least three / *at most three diamonds.
→ Alice has 2 or less / 4 or more diamonds. ✗

⋆ In the antecedent of a conditional:

(29) If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds, she wins.

⋆ In the restriction of a universal:

(30) Everyone who has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds wins.

What is GQT missing?

entailments	scalar implicatures	ignorance	accept in DE env
✓	✓+ ?	?	?

Sketch of the solution:

⋆ Keep the GQT way of getting entailments.
⋆ Keep scalar implicatures.
⋆ Add domain alternatives [Kennedy, 2015, Spector, 2015].
⋆ Make the domain alternatives of SMs obligatory [Spector, 2015].
⋆ Try to derive rather than stipulate the number, type, and status of the alternatives in each case.

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Proposal: Truth conditions and presupposition

the numeral || [Link, 1983, Buccola and Spector, 2016]

$[\![\text{three}]\!] = 3$ $\qquad$ $[\![\text{is}_{\text{Card}}]\!]\,(3) = \lambda x\,.\,|x| = 3$

much/little || [Seuren, 1984, Kennedy, 1997]

$[\![\text{much}]\!]\,(n) = \lambda d\,.\,d \leq n$ $\qquad$ $[\![\text{little}]\!]\,(n) = \lambda d\,.\,d \geq n$

truth conditions || [Krifka, 1999, Von Stechow, 2005, Heim, 2007, Hackl, 2009]

$$\begin{aligned}
(\exists\ (\text{n P}))(\text{Q}) &= 1 \text{ iff } \exists x[|x| = \underline{n \wedge P(x) \wedge Q(x)]} \\
[\text{comp}(\text{much/little})](\text{n})(\text{P})(\text{Q}) &= 1 \text{ iff } |P \cap Q| \in [\![\text{much/little}]\!]\,(n) \\
[\text{sup}(\text{much/little})](\text{n})(\text{P})(\text{Q}) &= 1 \text{ iff } |P \cap Q| \in [\![\text{much/little}]\!]\,(n)
\end{aligned}$$

the presupposition of [sup] || [Hackl, 2009, Gajewski, 2010]

$$|\,[\![\text{much/little}]\!]\,(n)| \geq 2$$

✓ Entailments

(31) *3 P Q:*

$\exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \Rightarrow |P \cap Q| \geq 3$ (l.b.)

(32) *more than 3 P Q:*

$|P \cap Q| \in \overline{[\![\text{much}]\!]\,(3)} \Leftrightarrow |P \cap Q| \in \{4, 5, \dots\}$ (l.b.)

(33) *less than 3 P Q:*

$|P \cap Q| \in \overline{[\![\text{little}]\!]\,(3)} \Leftrightarrow |P \cap Q| \in \{\dots, 0, 1, 2\}$ (u.b.)

(34) *at most 3 P Q:*

$|P \cap Q| \in [\![\text{much}]\!]\,(3) \Leftrightarrow |P \cap Q| \in \{\dots, 0, 1, 2, 3\}$ (u.b.)

(35) *at least 3 P Q*:

$|P \cap Q| \in [\![\text{little}]\!]\,(3) \Leftrightarrow |P \cap Q| \in \{3, 4, \dots\}$ (l.b.)

Proposal: Alternatives

Scalar alternatives can be obtained by replacing n in the numeral argument with its scalar alternatives (other numerals)

BNs: $\{\exists x[|x| = m \wedge P(x) \wedge Q(x)] : \ m \in S\}$

CMs: $\{|P \cap Q| \in \overline{[\![\text{much/little}]\!]\,(m)} : \ m \in S\}$

SMs: $\{|P \cap Q| \in [\![\text{much/little}]\!]\,(m) : \ m \in S\}$

Domain alternatives can be obtained by replacing the whole numeral argument with its subsets

BNs: NA (the numeral argument is just a degree)

CMs: $\{|P \cap Q| \in A : \ A \subseteq \overline{[\![\text{much/little}]\!]\,(n)}\}$

SMs: $\{|P \cap Q| \in A : \ A \subseteq [\![\text{much/little}]\!]\,(n)\}$

active by presup!

Scalar alternatives

$$
\begin{aligned}
&\text{ScalAlts}(\textit{3 P Q})\\
&= \text{ScalAlts}(\exists x[|x| = 3 \land P(x) \land Q(x)])\\
&= \{\ldots, \exists x[|x| = 2 \land P(x) \land Q(x)], \exists x[|x| = 4 \land P(x) \land Q(x)], \ldots\}\\
&= \{\ldots, \textit{2 P Q}, \textit{4 P Q}, \ldots\}
\end{aligned}
$$

$$
\begin{aligned}
&\text{ScalAlts}(\textit{more/less than 3 P Q})\\
&= \text{ScalAlts}(|P \cap Q| \in \overline{[\![\text{much/little}]\!]\,(3)})\\
&= \{\ldots, |P \cap Q| \in \overline{[\![\text{much/little}]\!]\,(2)}, |P \cap Q| \in \overline{[\![\text{much/little}]\!]\,(4)}, \ldots\}\\
&= \{\ldots, \textit{more/less than 2 P Q}, \textit{more/less than 4 P Q}, \ldots\}
\end{aligned}
$$

$$
\begin{aligned}
&\text{ScalAlts}(\textit{at most/least 3 P Q})\\
&= \text{ScalAlts}(|P \cap Q| \in [\![\text{much/little}]\!]\,(3))\\
&= \{\ldots, |P \cap Q| \in [\![\text{much/little}]\!]\,(2), |P \cap Q| \in [\![\text{much/little}]\!]\,(4), \ldots\}\\
&= \{\ldots, \textit{at most/least 2 P Q}, \textit{at most/least 4 P Q}, \ldots\}
\end{aligned}
$$

Subdomain alternatives

SubDomAlts(*3 P Q*): NA

SubDomAlts(*more/less than 3 P Q*)
$= \text{SubDomAlts}(|P \cap Q| \in [\![\text{much/little}]\!]\,(3))$
$= \text{SubDomAlts}(|P \cap Q| \in \{4, 5, \ldots\}/\{0, 1, 2\})$
$= \{|P \cap Q| \in \{4\}, |P \cap Q| \in \{4, 7\}, \ldots\}$ / $\{|P \cap Q| \in \{0\},$
$|P \cap Q| \in \{0, 1\}, \ldots\}$

SubDomAlts(*at most/least 3 P Q*)
$= \text{SubDomAlts}(|P \cap Q| \in [\![\text{much/little}]\!]\,(3))$
$= \text{SubDomAlts}(|P \cap Q| \in \{0, 1, 2, 3\}/\{3, 4, \ldots\})$
$= \{|P \cap Q| \in \{0\}, |P \cap Q| \in \{1, 3\}, \ldots\}$ / $\{|P \cap Q| \in \{3\},$
$|P \cap Q| \in \{4, 8\}, \ldots\}$

active by presup!

Proposal: Implicature calculation system

[Chierchia, 2013]

O	to exhaustify the scalar alternatives of BNs, CMs, and SMs

(36) $[\![O_{ALT}(\phi)]\!]^{g,w} = [\![\phi]\!]^{g,w} \wedge \forall p \in [\![\phi]\!]^{ALT}\ [p \rightarrow \lambda w' . [\![\phi]\!]^{g,w'} \subseteq p]$

O^{PS}	to exhaustify the subdomain alternatives of CMs and SMs

(37) $O^{PS}_{ALT}(\phi)$ is defined iff $O^{S}_{ALT}(\phi) \subset \phi$.
Whenever defined, $O^{PS}_{ALT}(\phi) = O^{S}_{ALT}(\phi)$,
where

a. $O^{S}_{ALT}(\phi_w) = \phi_w \wedge \forall p \in ALT\ [\pi(p)_w \rightarrow \pi(\lambda w . \phi_w) \subseteq \pi(p)]$,
where

(i) $\pi(q) = {}^{\alpha}q \wedge {}^{\pi}q$.

$\boxdot$	last resort, silent, matrix-level, universal doxastic modal

✓ Implicatures from scalar alternatives

considering only alternatives that do not lead to the problematic 'exactly' meanings

⋆ Unembedded:

(38) $O_{ScalAlts}$ *(Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.)*
$\rightsquigarrow$ ¬ Alice has 4 / more than 5 / less than 1 / at most 1 / at least 5 diamonds.

implicatures

⋆ In the scope of a universal operator:

(39) $O_{ScalAlts}$ *(Alice is required to have 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.)*
$\rightsquigarrow$ ¬ Alice is required to have 4 / more than 4 / less than 2 / at most 2 / at least 4 diamonds.

implicatures

✓ Implicatures from scalar alternatives

considering only alternatives that do not lead to the problematic 'exactly' meanings

- ⋆ In the antecedent of a conditional:

(40) $O_{ScalAlts}$ *(If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds she wins.)*
$\rightsquigarrow$ ¬ If Alice has 2 / more than 2 / less than 4 / at most 4 / at least 2 diamonds she wins.

$\boxed{\text{implicatures}}$

- ⋆ In the scope of negation:

(41) $O_{ScalAlts}$ *(Alice doesn't have 3 / more than 3 / less than 3 / *at most 3 / *at least 3 diamonds.)*
$\rightsquigarrow$ ¬ Alice doesn't have 1 / more than 1 / less than 5 / at most 5 / at least 1 diamonds.

$\boxed{\text{implicatures}}$

✓ Implicatures from subdomain alternatives

⋆ Unembedded:

(42) Alice has more/less than 3 / at most/least 3 diamonds.

a. $|P \cap Q| \in D$ $\boxed{\text{no } O^{PS}_{SubDomAlts}\ !}$

b. $O^{PS}_{SubDomAlts}\,(|P \cap Q| \in D)$
$= |P \cap Q| \in D \wedge \neg(|P \cap Q| \in A) \wedge \neg(|P \cap Q| \in B) \ldots$, for all $A, B, \cdots \subset D$, $= \bot$ $\boxed{\text{contradiction!}}$

c. $O^{PS}_{SubDomAlts}\ \boxdot\,(|P \cap Q| \in D)$
$= \boxdot|P \cap Q| \in D \wedge \neg\boxdot\,(|P \cap Q| \in A) \wedge \neg\boxdot\,(|P \cap Q| \in B) \ldots$,
for all $A, B, \cdots \subset D$ $\boxed{\text{ignorance implicatures}}$

⋆ Ignorance optional for CMs, obligatory for SMs.

✓ Implicatures from subdomain alternatives

clash with ‘exactly’-inducing implicature from scalar alternatives!

(43) Alice has more than 2 / at least 3 diamonds.

$$O^{PS}_{SubDomAlts} \boxdot O_{ScalAlts} (|P \cap Q| \in \{3, 4, \ldots\})$$

$$= \underline{\boxdot O_{ScalAlts} (|P \cap Q| \in \{3, 4, \ldots\})} \wedge \underline{\neg \boxdot (|P \cap Q| \in \{3\})} \wedge \underline{\neg \boxdot (|P \cap Q| \in \{4, 7\})} \wedge \underline{\neg \ldots}$$

$$= \underline{\boxdot (|P \cap Q| \in \{3\})} \wedge \underline{\neg \boxdot (|P \cap Q| \in \{3\})} \wedge \underline{\neg \boxdot (|P \cap Q| \in \{4, 7\})} \wedge \underline{\neg \ldots}$$

$$= \bot$$

- ⋆ Prune offending SubDomAlts? That would violate $O^{PS}_{SubDomAlts}$, so no. ✗
- ⋆ Prune offending ScalAlt? ✓

✓ Implicatures from subdomain alternatives

⋆ In the scope of a universal operator:

(44) Alice is required to have more/less than 3 / at most/least 3 diamonds.

a. $\Box(|P \cap Q| \in D)$ $\boxed{\text{no } O^{PS}_{SubDomAlts}\ !}$

b. $\Box\ O^{PS}_{SubDomAlts}\ (|P \cap Q| \in D)$ $\boxed{\text{contradiction!}}$

c. $O^{PS}_{SubDomAlts}\ (\Box(|P \cap Q| \in D))$ $\boxed{\text{non-ignorance implicatures}}$

d. $O^{PS}_{SubDomAlts}\ (|P \cap \Box Q| \in D)$ $\boxed{\text{contradiction!}}$

e. $O^{PS}_{SubDomAlts}\ (\boxdot(|P \cap \Box Q| \in D))$ $\boxed{\text{ignorance implicatures}}$

⋆ Ignorance optional for both CMs and SMs.

✓ Implicatures from subdomain alternatives

- In the scope of negation:

(45) Alice doesn't have more/less than 3 / *at most/least 3 diamonds.

a. $\neg(|P \cap Q| \in D)$ — $\boxed{\text{no } O^{PS}_{SubDomAlts}\text{ !}}$

b. $\neg O^{PS}_{SubDomAlts}\ (|P \cap Q| \in D)$ — $\boxed{\text{contradiction!}}$

c. $O^{PS}_{SubDomAlts}\ \neg(|P \cap Q| \in D)$
$= \neg(|P \cap Q| \in D)$ — $\boxed{\text{no proper strengthening!}}$

d. $O^{PS}_{SubDomAlts}\ \boxdot\neg(|P \cap Q| \in D)$
$= \boxdot\neg(|P \cap Q| \in D)$ — $\boxed{\text{no proper strengthening!}}$

- No ignorance implicatures sanctioned formally.

✓ Acceptability in DE environments

⋆ In the scope of negation:

(46) Alice doesn't have more/less than three / *at most/least three diamonds.

a. $\neg(|P \cap Q| \in D\})$ — $\boxed{\text{no } O^{PS}_{SubDomAlts}\ !}$

b. $\neg O^{PS}_{SubDomAlts}\ (|P \cap Q| \in D)$ — $\boxed{\text{contradiction!}}$

c. $O^{PS}_{SubDomAlts}\ \neg(|P \cap Q| \in D)$ — $\boxed{\text{no proper strengthening!}}$

d. $O^{PS}_{SubDomAlts}\ \boxdot\neg(|P \cap Q| \in D)$ — $\boxed{\text{no proper strengthening!}}$

⋆ CMs can be parsed as in (a). No parsing option for SMs.

✓ Acceptability in DE environments

⋆ In the antecedent of a conditional / restriction of a universal:

(47) Everyone who has more/less than 3 / at most/least 3 diamonds wins.

$$\begin{array}{ccc} \forall x[\#\text{ di } x \text{ has } \in D \rightarrow \ldots] & \wedge & \exists x[\#\text{ of di } x \text{ has } \in D] \\ \Downarrow & & \Uparrow \\ \forall x[\#\text{ di } x \text{ has } \in D' \rightarrow \ldots] & \wedge & \exists x[\#\text{ of di } x \text{ has } \in D'] \end{array}$$

⋆ SubDomAlts not entailed, so they must be false.

⋆ However, negating them leads to contradiction.

⋆ We can rescue the parse with $\boxdot$.

⋆ Ignorance implicatures about the presupposition: The speaker is sure that here is someone such that the # of diamonds they have is in D, but not sure about any subsets of D.

Taking stock

entailments	scalar implicatures	ignorance	accept in DE env
✓	✓	✓	✓

Outline

Compositionality

Lexical entries for the numeral, *much*/*little*, [comp], and [sup] that

⋆ give us the right truth conditions and a natural way to derive the number, type, and status of the alternatives in each case;

⋆ link up naturally to meanings elsewhere;

⋆ ensure that the resulting bare or modified numeral DPs will pose no further compositional challenges, as they are generalized quantifiers.

Predicative uses

(48) The three / more/less than three / at most/least three NP

(49) We are three / more/less than three / at most/least three.

(50) Plant a tree every three houses.

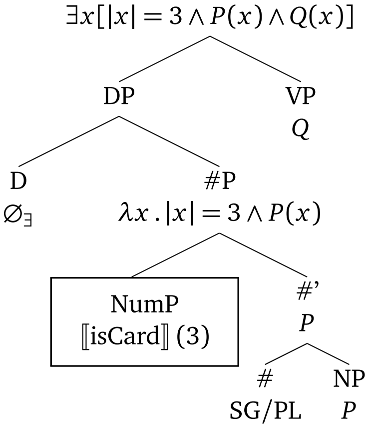
(51) If two relatives of mine die, I'll be rich.

⋆ Use [Partee, 1987]'s BE to typeshift the generalized quantifier meanings into predicative meanings:

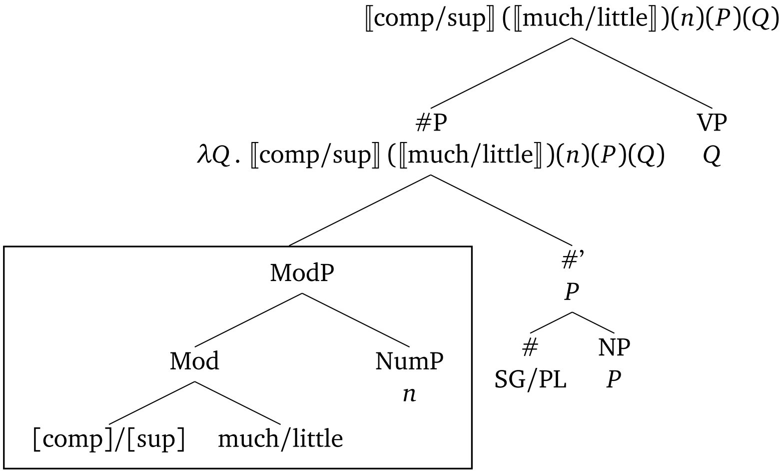
(52) $[\![\mathrm{BE}]\!] = \lambda Q_{\langle \alpha t,t\rangle} . \lambda x_\alpha . Q(\lambda y_\alpha . y = x)$

(53) $[\![\mathrm{BE}]\!]\,([\![\text{at most three students}]\!])$
$$= [\lambda Q_{\langle et,t\rangle} . \lambda x_e . Q(\lambda y_e . y = x)](\lambda Q_{\langle e,t\rangle} . |P \cap Q| \in [\![\text{much}]\!]\,(3))$$
$$= \lambda x_e . [\lambda Q_{\langle e,t\rangle} . |P \cap Q| \in [\![\text{much}]\!]\,(3)](\lambda y_e . y = x)$$
$$= \lambda x_e . |P \cap \lambda y_e . y = x| \in [\![\text{much}]\!]\,(3)$$

Constituent structure



Constituent structure



Outline

Conclusion

- ⋆ A unified account of bare and modified numerals that builds conservatively on the original GQT account.

- ⋆ Derives their patterns w.r.t. entailments, scalar implicatures, ignorance, and acceptability in DE environments from their morphological pieces.

- ⋆ The account is more comprehensive, has better empirical coverage, and is less stipulative than previous accounts.

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