Modified numerals
between
polarity and valence

3
more/less than 3
at most/least 3

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Preview

Part 1:
A theory of numerals that gets the sensitivity to polarity of SMNs

Part 2:
Experiments confirming polarity data, bringing to light valence data

Part 3:
A theory of numerals that gets the sensitivity to valence of SMNs

Outline

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Generalized Quantifier Theory

(1) $\quad [\![\text{every}]\!] = \lambda P . \lambda Q . P \subseteq Q$

(2) $\quad [\![\text{no}]\!] = \lambda P . \lambda Q . P \cap Q = \varnothing$

(3) $\quad [\![\text{a}]\!] = \lambda P . \lambda Q . P \cap Q \neq \varnothing$

(4) $\quad [\![\text{three}]\!] = \lambda P . \lambda Q . |P \cap Q| \geq 3$

(5) $\quad [\![\text{more than three}]\!] = \lambda P . \lambda Q . |P \cap Q| > 3$

(6) $\quad [\![\text{less than three}]\!] = \lambda P . \lambda Q . |P \cap Q| < 3$

(7) $\quad [\![\text{at least three}]\!] = \lambda P . \lambda Q . |P \cap Q| \geq 3$

(8) $\quad [\![\text{at most three}]\!] = \lambda P . \lambda Q . |P \cap Q| \leq 3$

(9) $\quad [\![\text{exactly three}]\!] = \lambda P . \lambda Q . |P \cap Q| = 3$

(10) $\quad [\![\text{between three and five }]\!] = \lambda P . \lambda Q . 3 \leq |P \cap Q| \leq 5$

✓ Entailments [Horn, 1972, van Benthem, 1986]

⋆ *3 / more than 3 / at least 3* have lower-bounding entailments, but *less than 3 / at most 3* have upper-bounding entailments:

(11) a. Alice has 3 / more than 3 / at least 3 diamonds.
   b. ¬ The number of diamonds that Alice has is 2 or less / 3 or less / 2 or less.
   c. Alice has 3 / more than 3 / at least 3 diamonds, # if not less.

(12) a. Alice has less than 3 / at most 3 diamonds.
   b. ¬ The number of diamonds that Alice has is 3 or more / 4 or more.
   c. Alice has less than 3 / at most 3 diamonds, # if not more.

⋆ GQT effortlessly gets this.

✓ The upper bound of BNs [Horn, 1972, Spector, 2013]

⋆ BNs carries also carries an upper-bounding inference, which is however optional:

(13) a. Alice has 3 diamonds.
    b. ¬ The number of diamonds that Alice has is 4 or more.
    c. Alice has 3 diamonds, if not more.

⋆ Idea: The upper-bounding inference is an implicature. [Horn, 1972]

⋆ GQT + this adjustment gets this.

✗ The scalar implicatures of CMNs and SMNs [Krifka, 1999]

⋆ This view predicts scalar alternatives for CMNs and SMNs also:

(14) a. ScalAlts(*3 P Q*)
    = {..., *2 P Q, 4 P Q,* ...}
  b. ScalAlts(*more/less than 3 P Q*)
    = {..., *more/less than 2 P Q, more/less than 4 P Q,* ...}
  c. ScalAlts(*at most/least 3 P Q*)
    = {..., *at most/least 2 P Q, at most/least 4 P Q,* ...}

⋆ But CMNs and SMNs look like they don't have them ...

(15) Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
    ⤳ ¬ Alice has 4 / *more than 4 / *less than 2 / *at most 2 / *at least 4 diamonds.
    (Total predicted meaning: She has exactly 3 / exactly 4 / exactly 2 / exactly 3 / exactly 3 diamonds.)

✗ The ignorance inferences of SMNs [Geurts and Nouwen, 2007]

⋆ SMNs give rise to strong speaker ignorance inferences:

(16) A hexagon has 6 / more than 5 / ??at least 6 sides.

(17) I have 3 / more than 2 / ??at least 3 children.

(18) This plane has more than 5 / ??at least 6 emergency exits.

⋆ GQT doesn't predict this at all.

✗ The unacceptability of SMNs in DE env's [Nilsen, 2007]

⋆ BNs and CMNs can be interpreted under negation, but SMNs can't:

(19) Alice doesn't have 3 / more than 3 / less than 3 diamonds.
    → Alice has 2 or less / 3 or less / 3 or more diamonds.   ✓

(20) Alice doesn't have *at least three / *at most three diamonds.
    → Alice has 2 or less / 4 or more diamonds.   ✗

⋆ GQT doesn't predict this at all.

Conclusion:

⋆ GQT is too uniform, doesn't capture a lot of differences among BNs, CMNs, and SMNs.

Response:

⋆ Non-uniform analyses, designed to capture the differences.

Outline

Truth conditions / entailments

★ BNs: adjectival meaning. [Link, 1987, Krifka, 1999]

(21) $\llbracket \text{three} \rrbracket$
$= \lambda x . |x| = n$ or
$= \lambda P . \lambda x . |x| = n \wedge P(x)$

★ CMNs/comparative semantics: $\langle dt, t \rangle$, max. [Hackl, 2000]

(22) $\llbracket \text{more/less than three} \rrbracket$
$= \lambda D_{\langle d,t \rangle} . max(\lambda d . D(d)) > / < 3$
$\llbracket \text{more than three students smiled} \rrbracket$
$= max(\lambda n . \exists x [|x| = n \wedge \text{students}(x) \wedge \text{smiled}(x)]) > / < 3$

In what follows I will adopt both (in slightly modified forms).

Note: I believe the GQT-style way at getting at the cardinality directly via a cardinality function would work also, although, of course, the compositionality would look different.

Scalar implicatures, ignorance, acceptability in DE env's

* ⋆ Very numerous and very diverse approaches. [Krifka, 1999, Fox and Hackl, 2006, Geurts and Nouwen, 2007, Büring, 2008, Nouwen, 2010, Mayr, 2013, Coppock and Brochhagen, 2013, Cohen and Krifka, 2014, Kennedy, 2015, Spector, 2015]

* ⋆ Empirically, a goal to derive
  - no scalar implicatures for CMNs and/or SMNs;
  - ignorance for SMNs; and
  - badness under negation for SMNs.

* ⋆ Conceptually, a trend towards saying
  - that BNs may or may not have scalar alternatives but CMNs and SMNs don't (or they do, but they are neutralized);
  - that SMNs have a disjunctive form and their alternatives are the individual disjuncts (domain alternatives); and
  - that these alternatives of SMNs are obligatory and cannot be used vacuously (must lead to strengthening).

There are issues with both.

Empirical issues: Scalar implicatures I

* ⋆ CMNs and SMNs do give rise to direct scalar implicatures.

(23) *Context: Grades are attributed on the basis of the number of problems solved. People who solve between 1 and 5 problems get a C. People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.*
John solved more than 5 problems. Peter solved more than 9.
⤳ ¬ John solved more than 9.   [Spector, 2014]

(24) Alice is required to have 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
⤳ ¬ Alice is required to have 4 / more than 4 / less than 2 / at most 2 / at least 4 diamonds.

Note: The universal case is often acknowledged, although it is usually not derived from the traditional scalar alternatives.

Empirical issues: Scalar implicatures II

⋆ CMNs and SMNs also give rise to indirect scalar implicatures:

(25) If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds she wins.
⤳ ¬ If Alice has 2 / more than 2 / less than 4 / at most 4 / at least 2 diamonds she wins.

Note: Approaches that depart from the traditional scalar alternatives for BNs, CMNs, and SMNs are typically unable to derive this.

Empirical issues: Scalar implicatures III

* BNs sometimes pose a challenge also:

(26) Alice doesn't have 3 / more than 3 / less than 3 / *at most 3
/ *at least 3 diamonds.
⤳ ¬ Alice doesn't have *2 / *more than 2 / *less than 4 / *at
most 4 / *at least 2 diamonds.
(Total predicted meaning: She has exactly 2 / exactly 3 /
exactly 3 / exactly 4 / exactly 2 diamonds.)

* But, with coarser granularity, the implicatures seem just right:

(27) Alice doesn't have 3 / more than 3 / less than 3 / *at most 3
/ *at least 3 diamonds.
⤳ ¬ Alice doesn't have 1 / more than 1 / less than 5 / at
most 5 / at least 1 diamond(s).

Empirical issues: Scalar implicatures IV

I will argue
* that BNs, CMNs, and SMNs can all in principle give rise to scalar implicatures; and
* that the overgenerated scalar implicatures
  - are not obviously wrong;
  - may be dispreferred due to granularity/ competition with a bare numeral; and   [Cummins et al., 2012, Enguehard, 2018]
  - in the case of CMNs and SMNs in unembedded contexts may be further dispreferred due to a clash with ignorance (to be clarified soon).

Empirical issues: Ignorance I

⋆ The ignorance contrast is more nuanced than has been assumed.

⋆ BNs are not compatible with an ignorant speaker, but CMs and SMs are:

(28) a. I don't know how many diamonds Alice has, #but she has 3.
   b. I don't know how many diamonds Alice has, but she has more than 3 / less than 3 / at most 3 / at least 3.

⋆ CMs compatible with exact knowledge but SMs not. [Nouwen, 2015]

(29) a. There were exactly 62 mistakes in the manuscript, so that's more than 50.
   b. There were exactly 62 mistakes in the manuscript, #so that's at least 50.

Empirical issues: Ignorance II

I will argue

⋆ that in unembedded contexts BNs don't give rise to ignorance implicatures but CMNs and SMNs both do, although the latter more strongly than the former (cancellable vs. non-cancellable ignorance, can be strong in both depending on QUD); and

[Cremers et al., 2017]

⋆ the same basic mechanisms are involved in deriving ignorance for CMNs as for SMNs (domain alternatives), and they are also responsible for other parallel (and similarly graded) effects in CMNs and SMNs such as quantificational variability effects.

[Alexandropoulou et al., 2016]

Empirical issues: Acceptability in DE environments

⋆ SMNs are not only systematically bad under negation but also systematically good in other types of DE environments such as the antecedent of a conditional or the restriction of a universal:

(30) If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds, she wins.

(31) Everyone who has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds wins.

I will argue
⋆ that the same mechanism derives both unacceptability under negation and acceptability in the antecedent of a conditional and the restriction of a universal, and   [Spector, 2014]
⋆ the opposite patterns are due to the fact that the mechanism is sensitive to an empirical difference between these two types of environments, namely, the fact that the latter carry an existential presupposition.   [Spector, 2014, Nicolae, 2017]

Conceptual issues: Stipulations about the alternatives

⋆ Why should BNs have scalar alternatives but CMNs and SMNs not?

⋆ Why should only SMNs have domain alternatives, and why should they be obligatory?

I will argue

⋆ that BNs, CMNs, and SMNs all have scalar alternatives due to the numeral in their meaning;

⋆ that CMNs and SMNs both have subdomain alternatives because the *much/little* in their meaning creates a domain of degrees around the numeral; and

⋆ that the subdomain alternatives of SMNs are obligatory because they are always active due to the domain-size related presupposition of the superlative morpheme in their meaning.

Outline

Proposal: Truth conditions and presupposition

| the numeral | ‖ ‖ [Link, 1983, Buccola and Spector, 2016] |

$[\![n]\!] = n$ $[\![\text{is}_{\text{Card}}]\!](n) = \lambda x . |x| = n$

| much/little | ‖ ‖ [Seuren, 1984, Kennedy, 1997] |

$[\![\text{much}]\!](n) = \lambda d . d \leq n$ $[\![\text{little}]\!](n) = \lambda d . d \geq n$

| truth conditions (to trees) ‖ ‖ [Krifka, 1999, Heim, 2007, Hackl, 2000, Kennedy, 2015] |

$(\exists \ (n \ P))(Q)$ $\exists x[|x| = n \wedge P(x) \wedge Q(x)]$

$[\text{comp(much/little)}(n)]_1[(\exists((t_1)(P))(Q))]$ $max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!](n)}$

$[\text{at-sup(much/little)}(n)]_1[(\exists((t_1)(P))(Q))]$ $max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!](n)$

| the presupposition of [sup] | ‖ ‖ [Hackl, 2009, Gajewski, 2010] |

$|[\![\text{much/little}]\!](n)| \geq 2$

✓ Entailments

(32) *3 P Q:*
$\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$
$\Rightarrow$ the # of P that Q ≥ 3

(lower bound)

(33) *more/less than 3 P Q:*
$max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![much/little]\!]} (3)$
$\Rightarrow$ the # of P that Q $\in \{4, 5, \ldots\}/\{\ldots, 0, 1, 2\}$

(lower/upper bound)

(34) *at most/least 3 P Q:*
$max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![much/little]\!] (3)$
$\Rightarrow$ the # of P that Q $\in \{\ldots, 0, 1, 2, 3\}/\{3, 4, \ldots\}$

(upper/lower bound)

Proposal: Alternatives

> Scalar alternatives: Replace domain-n with domain-m.

BNs: $\{\exists x[|x| = m \land P(x) \land Q(x)] : m \in S\}$

CMNs: $\{max(\lambda d\,.\,\exists x[|x| = d \land P(x) \land Q(x)]) \in \overline{[\![much/little]\!]\,(m)} : m \in S\}$

SMNs: $\{max(\lambda d\,.\,\exists x[|x| = d \land P(x) \land Q(x)]) \in [\![much/little]\!]\,(m) : m \in S\}$

> Subdomain alternatives: Replace domain-n with its subsets.

BNs: NA   (the numeral argument is just a degree)

CMNs: $\{max(\lambda d\,.\,\exists x[|x| = d \land P(x) \land Q(x)]) \in A : A \subseteq \overline{[\![much/little]\!]\,(n)}\}$

SMNs: $\{max(\lambda d\,.\,\exists x[|x| = d \land P(x) \land Q(x)]) \in A : A \subseteq [\![much/little]\!]\,(n)\}$

> active by presup!

Proposal: Implicature calculation system [Chierchia, 2013]

O	to exhaustify the scalar alternatives of BNs, CMNs, and SMNs

(35) $[\![O_{ALT}(p)]\!]^w = p_w \wedge \forall q \in ALT\ [q_w \to p \subseteq q]$

O^{PS}	to exhaustify the subdomain alternatives of CMNs and SMNs

(36) $[\![O^{PS}_{ALT}(p)]\!]^w$ is defined iff $O^S_{ALT}(p) \subset p$.
Whenever defined, $[\![O^{PS}_{ALT}(p)]\!]^w = [\![O^S_{ALT}(p)]\!]^w$,
where

   a. $[\![O^S_{ALT}(p)]\!]^w = \pi(p)_w \wedge \forall q \in ALT\ [\pi(q)_w \to \pi(p) \subseteq \pi(q)]$,
     where

     (i) $\pi(r) = {}^{\alpha}r \wedge {}^{\pi}r$,
        where

           ${}^{\alpha}r$ and ${}^{\pi}r$ are the assert. and the presup. comp. of
           r.

$\boxdot$	last resort, silent, matrix-level, universal doxastic modal

General shape of the solution

⋆ $O_{ScalAlts}$ yields Scalar implicatures, $O^{PS}_{SubDomAlts}$ yields Ignorance and Acceptability in DE environments.

⋆ In particular, $O^{PS}_{SubDomAlts}$ derives Ignorance with the help of $\boxdot$, and the DE patterns due to PS – due to its sensitivity to presuppositional content, it is able to produce both unacceptability under negation and acceptability in the antecedent of a conditional and the restriction of a conditional.

(37) Alice has 3 diamonds.

    a. $O_{ScalAlts}$ $(\exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \wedge)$
    $= \exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \wedge$
    $\neg \exists x[|x| = 4 \wedge P(x) \wedge Q(x)] \wedge \ldots$

(38) Alice has more/less than 3 diamonds.

    a. $O_{ScalAlts}$ $(max(\lambda d \,.\, \exists x[|x| = d \wedge \ldots]) \in \overline{[\![\text{much/little}]\!]\,(3)})$
    $= max(\lambda d \,.\, \exists x[|x| = d \wedge \ldots]) \in \overline{[\![\text{much/little}]\!]\,(3)} \wedge$
    $\neg \, max(\lambda d \,.\, \exists x[|x| = d \wedge \ldots]) \in \overline{[\![\text{much/little}]\!]\,(4/2)} \wedge \ldots$

(39) Alice has at most/least 3 diamonds.

    a. $O_{ScalAlts}$ $(max(\lambda d \,.\, \exists x[|x| = d \wedge \ldots]) \in [\![\text{much}]\!]\,(3))$
    $= max(\lambda d \,.\, \exists x[|x| = d \wedge \ldots]) \in [\![\text{much/little}]\!]\,(3) \wedge$
    $\neg \, max(\lambda d \,.\, \exists x[|x| = d \wedge \ldots]) \in [\![\text{much/little}]\!]\,(2/4) \wedge \ldots$

⋆ And so on. We can derive all the scalar implicatures we want.

⋆ Scalar implicatures restricted by granularity and potential clash
$O_{ScalAlts}$ and $O_{SubDomAlts}^{PS}$.

✓ Implicatures from SubDomAlts: Ignorance

(40) Alice has more/less than 3 / at most/least 3 diamonds.

a. $O^{PS}_{SubDomAlts} (max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in D)$
$= max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in D \wedge$
$\neg \, max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in A) \wedge$
$\neg \, max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in B \wedge \ldots,$ for all $A, B, \cdots \subset D,$
$= \bot$ $\boxed{\text{contradiction}}$

b. $O^{PS}_{SubDomAlts} (\Box max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in D)$
$= \Box max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in D \wedge$
$\neg \, \Box \, max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in A \wedge$
$\neg \, \Box \, max(\lambda d \,.\, \exists x[|x| = d \ldots]) \in B \wedge \ldots,$ for all $A, B, \cdots \subset D$
          $\boxed{\text{ignorance implicatures}}$

⋆ The only consistent $O^{PS}_{SubDomAlts}$ parse is the ignorance parse. SMNs can only have this parse, hence obligatory ignorace. CMNs can also have a parse without $O^{PS}_{SubDomAlts}$, hence optional ignorance.
⋆ Similar reasoning for cases with an overt □ operator (but: for such cases, multiple consistent parsing possibilities).

✓ Implicatures from SubDomAlts: Negation

(41) Alice doesn't have more/less than three / *at most/least three diamonds.

a. $\neg O^{PS}_{SubDomAlts} (max(\lambda d . \exists x[|x| = d \dots]) \in D)$

> $O^{PS}_{SubDomAlts}$ leads to contradiction!

b. $O^{PS}_{SubDomAlts} (\neg max(\lambda d . \exists x[|x| = d \dots]) \in D)$

> $O_{SubDomAlts}$ vacuous, so PS violated!

c. $O^{PS}_{SubDomAlts} (\Box \neg max(\lambda d . \exists x[|x| = d \dots]) \in D)$

> $O_{SubDomAlts}$ vacuous, so PS violated!

⋆ All $O^{PS}_{SubDomAlts}$ parses fail. No parsing option for SMNs, hence infelicity under negation. CMNs can be parsed without $O^{PS}_{SubDomAlts}$, hence no problem.

✓ Implicatures from SubDomAlts: AntCond/RestUniv

(42) Everyone who has more/less than 3 / at most/least 3 diamonds wins.

$$\forall x[\# \text{ di } x \text{ has } \in D \to \dots] \quad \wedge \quad \exists x[\# \text{ of di } x \text{ has } \in D]$$
$$\Downarrow \qquad\qquad\qquad\qquad \Uparrow$$
$$\forall x[\# \text{ di } x \text{ has } \in D' \to \dots] \quad \wedge \quad \exists x[\# \text{ of di } x \text{ has } \in D']$$

⋆ SubDomAlts not entailed, so they must be false.

⋆ However, negating them leads to contradiction.

⋆ We can rescue the parse with $\boxdot$.

⋆ Ignorance implicatures about the presupposition: The speaker is sure that here is someone such that the # of diamonds they have is in D, but not sure about any D' ($D' \subset D$).

⋆ Thus there *is* a consistent $O^{PS}_{SubDomAlts}$ parse for SMNs, which is why they are felicitous in this type of DE environments.

Taking stock

⋆ As always, a lot of questions still remain, regarding:

- at-sup: We probably want to keep the extent analysis I offered but maybe the internal composition is different.

- PS: the presupposition of [sup] can help us explain why $O_{SubDomAlts}$ is obligatory for SMNs, but it doesn't explain why it should also come with PS; also, for CMNs PS is not crucial.

- $O^{PS}_{SubDomAlts}$ and $O_{ScalAlts}$: How should they interact?

⋆ Still, we have a unified account of bare and modified numerals that

- captures more patterns than previous accounts including, crucially, the acceptability of SMNs in DE environments; and

- derives them from general mechanisms involving exhaustification with O and the lexical meanings of BNs, CMNs, and SMNs.

Outline

Outline

Goal and expectations

⋆ Goal:

    To check the acceptability of CMNs and SMNs in the scope of negation and in the antecedent of a conditional / the restriction of a universal.

    To check how the combination of two DE operators (e.g., embedding in a negative antecedent of a conditional) affects acceptability.

⋆ Expectations:

    (Given our current assumptions so far) SMNs worse than CMNs under negation, but on a par with CMNs in the antecedent of a conditional / restriction of a universal.

    (Given what is claimed to be also the case for items that bear out the first expectation) CMNs and SMNs on a par in the negative antecedent of a conditional / restriction of a universal (or at least both highly rated).

Instructions

In this survey you will answer questions about a group of friends playing a game. At the beginning of the game each player gets dealt a hand of seven cards. After taking a quick look at them, they must place the cards face down and try to remember their hands. Then they take turns giving clues about their hands to the other players in the form of statements describing their hands. You will see what a player remembers about his/her cards and the statement s/he makes, then you will be asked if you think the other players will understand what s/he said.

Note: a ♣/♠ *or* ♦/♥ *a means that the player doesn't remember if a particular card in his hand was a diamond or a heart, or a club or a spade, respectively.*

Example trial

Charizard remembers:

Charizard says: I don't have at most 3 hearts.

Do you think the other players will understand what he said?

Yes.

No.

Summary of trials

[Modifier] = {more than, less than, at least, at most}
[Suit] = {diamonds, hearts, clubs, spades}

I have [Modifier] 3 [Suit]
I don't have [Modifier] 3 [Suit]
If you have [Modifier] 3 [Suit], then we have something in common
If you don't have [Modifier] 3 [Suit], then we have something in common
Everyone who has [Modifier] 3 [Suit] has something in common with me
Everyone who doesn't have [Modifier] 3 [Suit] has something in common with me

Participants and design

★ 99 native speakers of English recruited on MTurk
★ each participant saw all trials, presented in random order
★ each trial was obtained by crossing the following factors:
  Env (Decl, AntCond, RestUniv)
  x Polarity (Pos, Neg)
  x ModType (Comp, Sup)
  x ModMon (UE, DE)

Raw results by ModType
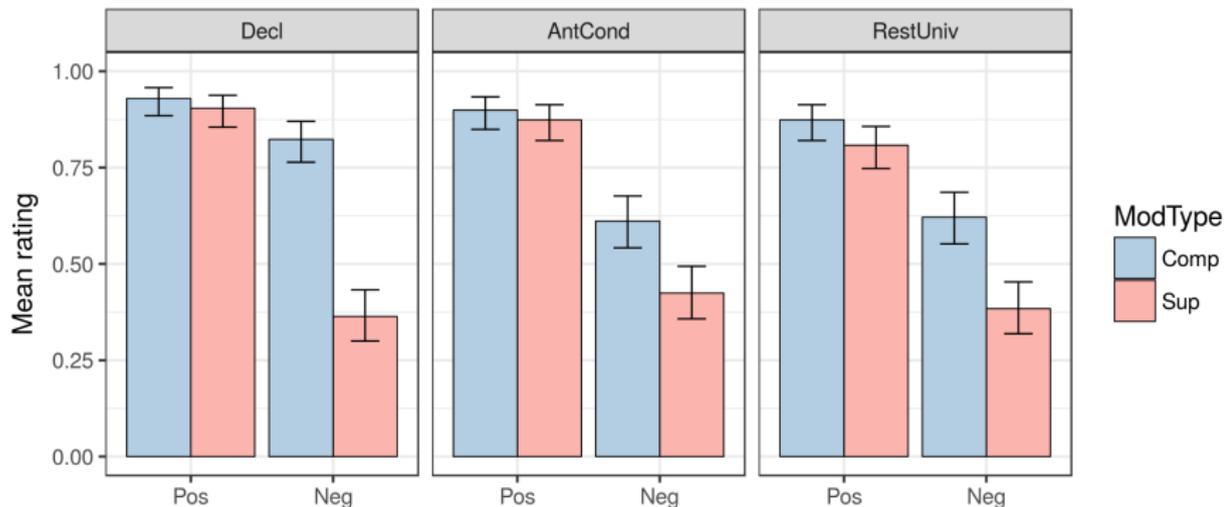
Bars represent 95% binomial confidence intervals.

Raw results by ModType crossed with ModMon

Bars represent 95% binomial confidence intervals.
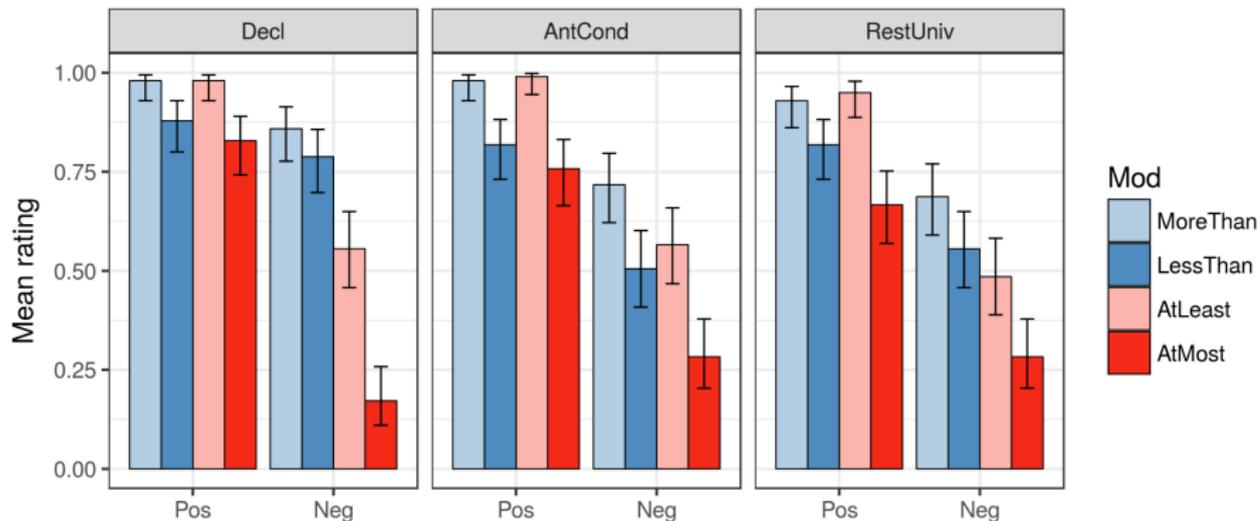
Statistical analysis

- ⋆ mixed-effects logistic regression models:
  response $\sim$ ModType * (ModMon *) Pol * Env
- ⋆ with the maximum random-effects structure that we could fit:
  $(1 + (ModType+Pol+Env)^2 \,|Subject)$
  $(1 + (ModType+ModMon+Pol+Env) \,|Subject)$
- ⋆ lsmeans contrasts at Pol = "Neg" from model by ModType:

Env	contrast	odds.ratio	CI	z.ratio	p.value
Decl	Comp - Sup	15.33	[6.49, 36.21]	7.602	<.0001
AntCond	Comp - Sup	2.61	[1.46, 4.67]	3.955	0.0001
RestUniv	Comp - Sup	3.43	[1.88, 6.25]	4.923	<.0001

Confidence level used: 0.95

Conf-level adjustment: bonferroni method for 3 estimates

Intervals are back-transformed from the log odds ratio scale

P value adjustment: holm method for 3 tests

Tests are performed on the log odds ratio scale

Expectations and what we found

⋆ SMNs worse than CMNs under negation, but on a par with CMNs in the antecedent of a conditional / restriction of a universal. ✓

⋆ CMNs and SMNs on a par in the negative antecedent of a conditional / restriction of a universal (or at least both highly rated). ✗

    - We found that SMNs are significantly worse.

    - SMNs in the antecedent of a conditional / restriction of a universal sensitive to whether the antecedent / restriction and the continuation match in positivity/negativity: [Cohen and Krifka, 2014]

(43) If you click at least twice, ...

    a. ... #the transaction will be canceled.

    b. ... you will get a prize.

(44) If you don't click at least twice, ...

    a. ... the transaction will be canceled.

    b. ... #you will get a prize.

    - Is this what went wrong for SMNs?

Outline

Goal and expectations

★ Goal:
   To check how the acceptability of CMNs and SMNs in the antecedent of a conditional / restriction of a universal varies depending on the polarity of the antecedent / restriction and the valence of the consequent / scope.

★ Expectations:
   SMNs significantly worse than CMNs in cases of mismatch but on a par with CMNs in cases of match.

Instructions

> *In this survey you will answer questions about a group of friends playing a game. At the beginning of the game each player gets dealt a hand of seven cards. They are not allowed to see their own cards but they are allowed to take a quick look at their neighbor's hand. They try to remember their neighbor's hand as well as they can because in the next step they have to come up with a rule that would make that neighbor (and possibly other players too) lose or win. You will see what a player remembers about their neighbor's hand and the rule they make up, then you will be asked if you think the other players will understand what they said. Note, we're not asking you if it is a good rule or a bad rule, but whether it is a rule that is going to be understandable for the other players to follow.*
>
> *Note: a ♣/♠ or ♦/♥ a means that the player doesn't remember if a particular card in his hand was a diamond or a heart, or a club or a spade, respectively.*

Example trial

Meowth remembers:
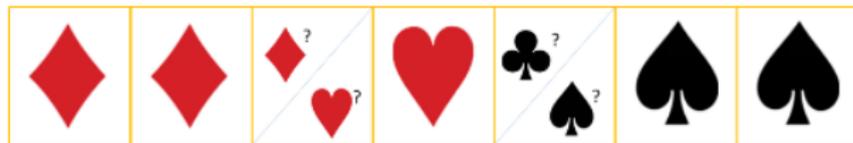
Meowth says: If you don't have at least 3 hearts, you lose.

Do you think the other players will understand what he said?

Yes.

No.

Summary of trials

[Modifier] = {more than, less than, at least, at most}
[Suit] = {diamonds, hearts, clubs, spades}

If you have [Modifier] 3 [Suit], you win
If you have [Modifier] 3 [Suit], you lose
If you don't have [Modifier]3 [Suit], you win
If you don't have [Modifier] 3 [Suit], you lose
Everyone who has [Modifier] 3 [Suit] wins
Everyone who has [Modifier] 3 [Suit] loses
Everyone who doesn't have [Modifier] 3 [Suit] wins
Everyone who doesn't have [Modifier] 3 [Suit] loses

Participants and design

- ★ 45 native speakers of English on Mturk; 5 excluded prior to analysis
- ★ each participant saw all trials, presented in random order
- ★ each trial was obtained by crossing the following factors:
  ModType (Comp, Sup) x ModMon (UE, DE)
  x Env (AntCond, RestUniv)
  x Pol (Pos, Neg)
  x Val (Pos, Neg)

Raw results by ModType
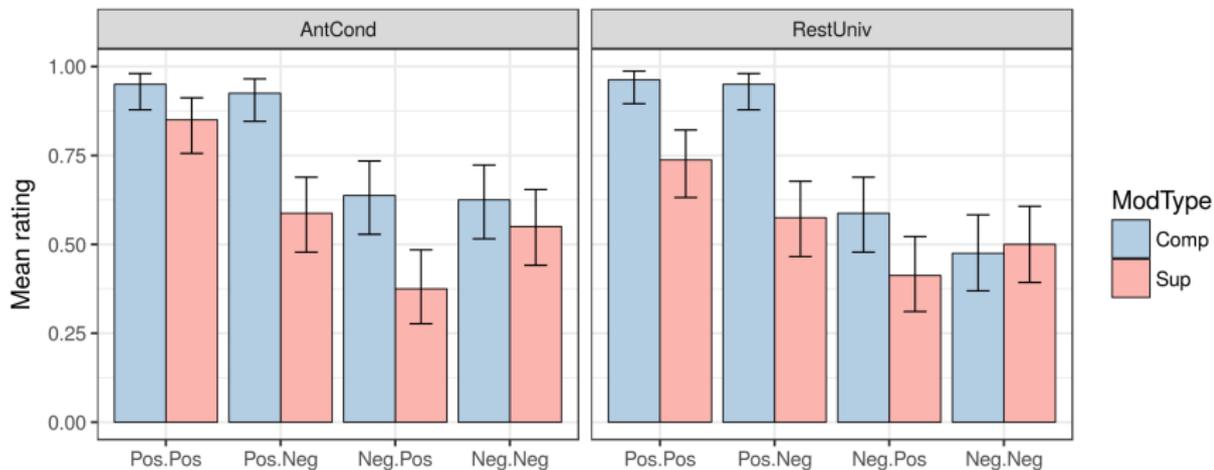
Bars represent 95% binomial confidence intervals.

Raw results by ModType crossed with ModMon

Bars represent 95% binomial confidence intervals.
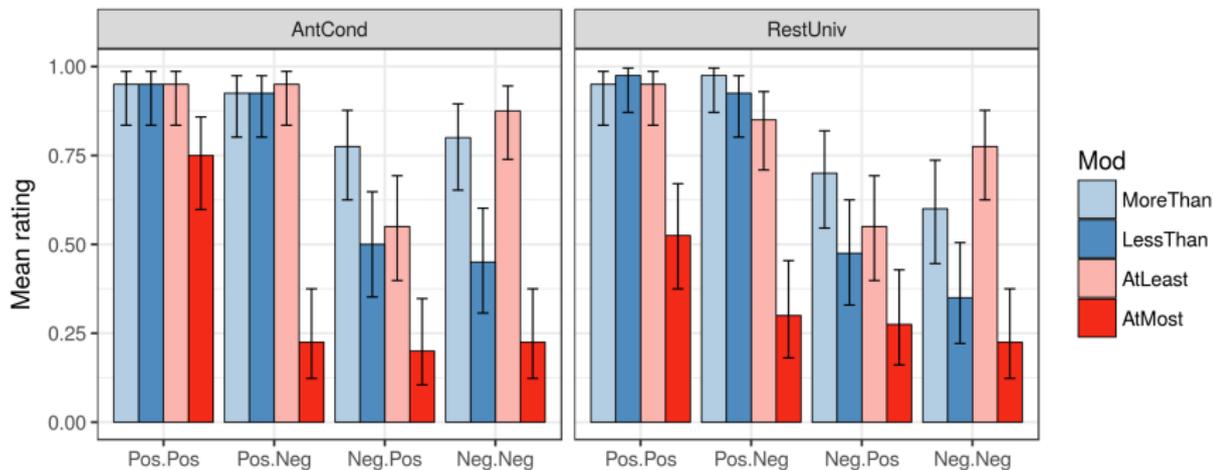
Expectations and what we found

⋆ SMNs significantly worse than CMNs in cases of mismatch but on a par with CMNs in cases of match. ✓/✗

    - If we look at ModType as a class, then ✓.

    - If we look at Mod, then ✗.

    - Also, very different patterns for ATLEAST vs. ATMOST.

Outline

Outline

The sources of valence

⋆ In our Exp 2 we tried very hard to not let the context play a role.
⋆ Maybe that's precisely what we need to do in order to understand what's going on.
⋆ Let's play with valence (properties positive or negative based on contextual assumptions).

Valence 1 and Valence 2, judgments

(45) a. at least, pos, pos
   If you solve at least three problems, you pass. ✓
   b. at least, pos, neg
   If you solve at least three problems, you fail. ✗
   c. at least, neg, pos
   If you make at least three mistakes, you pass. ✗
   d. at least, neg, neg
   If you make at least three mistakes, you fail. ?

(46) a. at most, pos, pos
   If you solve at most three problems, you pass. ✗
   b. at most, pos, neg
   If you solve at most three problems, you fail. ?
   c. at most, neg, pos
   If you make at most three mistakes, you pass. ✓
   d. at most, neg, neg
   If you make at most three mistakes, you fail. ✗

Valence 1 and Valence 2, judgment summary

Valence 1 = modifier + property 1, Valence 2 = property 2

modifier + property1	property2	judgment
at least + pos = pos	pos	✓
at least + pos = pos	neg	✗
at least + neg = neg	pos	✗
at least + neg = neg	neg	?
at most + pos = neg	pos	✗
at most + pos = neg	neg	?
at most + neg = pos	pos	✓
at most + neg = pos	neg	✗

$$\text{pos} + \text{pos} = ✓$$
$$\text{pos} + \text{neg} \,/\, \text{neg} + \text{pos} = ✗$$
$$\text{neg} + \text{neg} = ?$$

Outline

Solution: Exhaustification with E(ven), ScalAlts [Crnič, 2012]

E(ven) presupposes that its prejacent is the least likely among a set of scalar alternatives.

(47)*John read even one book.

John read one book $\prec_c$ John read two books ✗

(48) Even if John read one book, he will (still) pass the exam.

If John read one book, he will pass the exam $\prec_c$ If John read two books, he will pass the exam ✓

(49) If John read one book, *he will fail the exam.

If John read one book he will fail the exam $\prec_c$ If John read two books he will fail the exam ✗

(50) Even if John read all of the books, he will (still) fail the exam.

If John read all of the books, he will fail the exam $\prec_c$ If John read some of the books, he will fail the exam ✗

If John read $O_{ScalAlts}$ (all) (= all) of the books, he will fail the exam $\prec_c$ If John read $O_{ScalAlts}$ (some) (= some but not all) of the books, he will fail the exam ✓

Exhaustification with E(ven) *at least*

(51) **at least, pos, pos**
   If you solve at least three problems, you pass. ✓
   If you solve 3 problems you pass $\prec_c$ If you solve 4 problems you pass. ✓

(52) **at least, pos, neg**
   If you solve at least three problems, you fail. ✗
   If you solve 3 problems you fail $\prec_c$ If you solve 4 problems you fail. ✗

(53) **at least, neg, pos**
   If you make at least three mistakes, you pass. ✗
   If you make 3 mistakes you pass $\prec_c$ If you make 4 mistakes you pass. ✗

(54) **at least, neg, neg**
   If you make at least three mistakes, you fail. ?
   If you make 3 mistakes you fail $\prec_c$ If you make 4 mistakes you fail. ✓!

Exhaustification with E(ven) *at most*

(55) **at most, pos, pos**
  If you solve at most three problems, you pass. ✗
  If you solve 3 problems you pass $\prec_c$ If you solve 2 problems
  you pass. ✗

(56) **at most, pos, neg**
  If you solve at most three problems, you fail. ?
  If you solve 3 problems you fail $\prec_c$ If you solve 2 problems
  you fail. ✓

(57) **at most, neg, pos**
  If you make at most three mistakes, you pass. ✓
  If you make 3 mistakes you pass $\prec_c$ If you make 2 mistakes
  you pass. ✓

(58) **at most, neg, neg**
  If you make at most three mistakes, you fail. ✗
  If you make 3 mistakes you fail $\prec_c$ If you make 2 mistakes
  you fail. ✗

Exhaustification with E(ven)   *more than*

(59) **more than, pos, pos**
   If you solve more than three problems, you pass.   ✓
   If you solve 4 problems you pass $\prec_c$ If you solve 5 problems
   you pass. ✓

(60) **more than, pos, neg**
   If you solve more than three problems, you fail.   (✗)
   If you solve 4 problems you fail $\prec_c$ If you solve 5 problems
   you fail. ✗

(61) **more than, neg, pos**
   If you make more than three mistakes, you pass.   (✗)
   If you make 4 mistakes you pass $\prec_c$ If you make 5 mistakes
   you pass. ✗

(62) **more than, neg, neg**
   If you make more than three mistakes, you fail.   ✓
   If you make 4 mistakes you fail $\prec_c$ If you make 5 mistakes
   you fail. ✓

(63) **less than, pos, pos**
    If you solve less than three problems, you pass.   ✗
    If you solve 2 problems you pass $\prec_c$ If you solve 1 problem
    you pass. ✗

(64) **less than, pos, neg**
    If you solve less than three problems, you fail.   ✓
    If you solve 2 problems you fail $\prec_c$ If you solve 1 problem you
    fail. ✓

(65) **less than, neg, pos**
    If you make less than three mistakes, you pass.   ✓
    If you make 2 mistakes you pass $\prec_c$ If you make 1 mistake
    you pass.   ✓

(66) **less than, neg, neg**
    If you make less than three mistakes, you fail.   ✗
    If you make 2 mistakes you fail $\prec_c$ If you make 1 mistake you
    fail. ✗

Taking stock

⋆ Parallel effects with *more than* and *less than*, only infelicity judgments seem weaker.

⋆ E(ven) helps us account for valence effects.

⋆ It works only if we play with just a subset of the scalar alternatives. Essentially, we need to make the modified numerals be end of scale, whether low or high.

⋆ Moreover, it looks like we want to work with exact values. This will give rise to a non-monotonic scale where probability judgments rely exclusively on a contextual, non-entailment scale. We can get these exact values by having $O_{ScalAlts}$ inside the antecedent. Thus we get both a pre-exhaustified prejacent and pre-exhaustified alternatives (recall Crnic's solution for *all* associating with E across a DE operator).

⋆ But what about cases where we don't have just a negative property, but actual negation inside the antecedent?

Outline

Polarity and Valence, judgments

(67) a. **neg, at least, pos, pos**
If you don't solve at least three problems, you pass. ✗

   b. **neg, at least, pos, neg**
If you don't solve at least three problems, you fail. ✓

   c. **neg, at least, neg, pos**
If you don't make at least three mistakes, you pass. ?

   d. **neg, at least, neg, neg**
If you don't make at least three mistakes, you fail. ✗

(68) a. **neg, at most, pos, pos**
If you don't solve at most three problems, you pass. ?

   b. **neg, at most, pos, neg**
If you don't solve at most three problems, you fail. ✗

   c. **neg, at most, neg, pos**
If you don't make at most three mistakes, you pass. ✗

   d. **neg, at most, neg, neg**
If you don't make at most three mistakes, you fail. ?

Polarity and Valence, judgment summary

polarity + modifier + property1	property2	judgment
neg + at least + pos = neg	pos	✗
neg + at least + pos = neg	neg	✓
neg + at least + neg = pos	pos	?
neg + at least + neg = pos	neg	✗
neg + at most + pos = pos	pos	?
neg + at most + pos = pos	neg	✗
neg + at most + neg = neg	pos	✗
neg + at most + neg = neg	neg	?

derived-pos + pos = ?
pos + neg / neg + pos = ✗
derived-neg + neg = ✓ (*at least*: neg comes just from neg)
derived-neg + neg = ? (*at most*: neg comes from neg+*at most*+neg)

Exhaustification with E(ven)   *at least*

(69) a. **neg, at least, pos, pos**
   If you don't solve at least three problems, you pass.   ✗
   If you don't solve 3, you pass $\prec_c$ If you don't solve 2, you pass ✗

   b. **neg, at least, pos, neg**
   If you don't solve at least three problems, you fail.   ✓
   If you don't solve 3 problems, you fail $\prec_c$ If you don't solve 2 problems, you fail ✓

   c. **neg, at least, neg, pos**
   If you don't make at least three mistakes, you pass.   ?
   If you don't make 3 mistakes, you pass $\prec_c$ If you don't make 2 mistakes, you pass ✓

   d. **neg, at least, neg, neg**
   If you don't make at least three mistakes, you fail.   ✗
   If you don't make 3 mistakes, you fail $\prec_c$ If you don't make 2 mistakes, you fail ✗

Exhaustification with E(ven)   *at most*

(70) a. **neg, at most, pos, pos**
       If you don't solve at most three problems, you pass.   ?
       If you don't solve 3 problems, you pass $\prec_c$ If you don't
       solve 4 problems, you pass ✓

   b. **neg, at most, pos, neg**
       If you don't solve at most three problems, you fail.   ✗
       If you don't solve 3 problems, you fail $\prec_c$ If you don't solve
       4 problems, you fail ✗

   c. **neg, at most, neg, pos**
       If you don't make at most three mistakes, you pass.   ✗
       If you don't make 3 mistakes, you pass $\prec_c$ If you don't
       make 4 mistakes, you pass ✗

   d. **neg, at most, neg, neg**
       If you don't make at most three mistakes, you fail.   ?
       If you don't make 3 mistakes, you fail $\prec_c$ If you don't make
       4 mistakes, you fail ✓

Exhaustification with E(ven) *more than*

(71) a. **neg, more than, pos, pos**
   If you don't solve more than three problems, you pass.   ✗
   If you don't solve 4, you pass $\prec_c$ If you don't solve 3, you pass ✗

   b. **neg, more than, pos, neg**
   If you don't solve more than three problems, you fail.   ✓
   If you don't solve 3 problems, you fail $\prec_c$ If you don't solve 2 problems, you fail ✓

   c. **neg, more than, neg, pos**
   If you don't make more than three mistakes, you pass.   ?
   If you don't make 3 mistakes, you pass $\prec_c$ If you don't make 2 mistakes, you pass ✓

   d. **neg, more than, neg, neg**
   If you don't make more than three mistakes, you fail.   ✗
   If you don't make 3 mistakes, you fail $\prec_c$ If you don't make 2 mistakes, you fail ✗

(72) a. **neg, less than, pos, pos**
    If you don't solve less than three problems, you pass.   ?
    If you don't solve 3 problems, you pass $\prec_c$ If you don't
    solve 4 problems, you pass ✓

   b. **neg, less than, pos, neg**
    If you don't solve less than three problems, you fail.   ✗
    If you don't solve 3 problems, you fail $\prec_c$ If you don't solve
    4 problems, you fail ✗

   c. **neg, less than, neg, pos**
    If you don't make less than three mistakes, you pass.   ✗
    If you don't make 3 mistakes, you pass $\prec_c$ If you don't
    make 4 mistakes, you pass ✗

   d. **neg, less than, neg, neg**
    If you don't make less than three mistakes, you fail.   ✓
    If you don't make 3 mistakes, you fail $\prec_c$ If you don't make
    4 mistakes, you fail ✓

Taking stock

- ⋆ Again, parallel effects for *more than* and *less than*

Polarity and Valence, grand judgment summary

polarity + modifier + property1	property2	judgment
pos + at least + pos = pos	pos	✓
pos + at least + pos = pos	neg	✗
pos + at least + neg = neg	pos	✗
pos + at least + neg = neg	neg	?
pos + at most + pos = neg	pos	✗
pos + at most + pos = neg	neg	?
pos + at most + neg = pos	pos	✓
pos + at most + neg = pos	neg	✗
neg + at least + pos = neg	pos	✗
neg + at least + pos = neg	neg	✓
neg + at least + neg = pos	pos	?
neg + at least + neg = pos	neg	✗
neg + at most + pos = pos	pos	?
neg + at most + pos = pos	neg	✗
neg + at most + neg = neg	pos	✗
neg + at most + neg = neg	neg	?
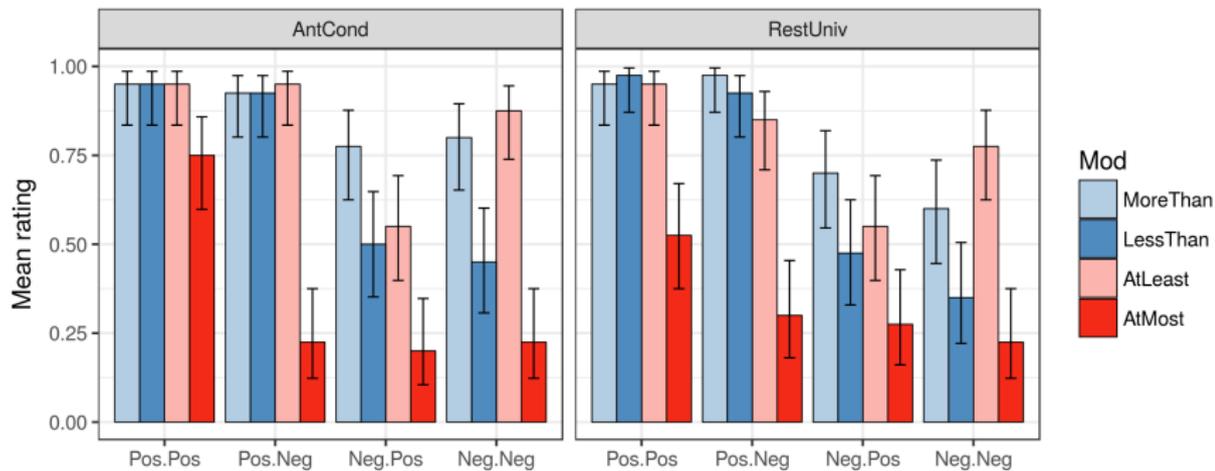
Exp 2 and expectations from polarity and valence I

⋆ I think the intrinsic positivity/negativity of the modifiers in combination with the *win*/*lose* continuations affect perceptions about the overall valence of the antecedent, resulting in different outcomes for *at least* and *at most*. Thus, what we labeled in the plot as Pos.Neg might not in fact always be neg+neg, and so on.

⋆ Consider Pos.Neg for *at least*: If you have at least diamonds, you lose. Thus, having *many* diamonds is bad. *at least* + neg = neg, which matches the neg in *lose*, which explains the ✓.

⋆ Consider Pos.Neg for *at most*: If you have at most three diamonds, you lose. Thus, having *few* diamonds is bad. *at most* + neg = pos, which does not match the neg in *lose*, which explains the ✗.

⋆ Consider Neg.Pos for *at least*: If you don't have at least three diamonds, you win. Thus, having *many* diamonds is bad. *at least* + neg = neg, which does not match the pos in *win*, which explains

Exp 2 and expectations from polarity and valence II

the ✗ (although note that it's not as deep a ✗ as for the previous case of mismatch).

⋆ Consider Neg.Pos for *at most*: If you don't have at most three diamonds, you win. Thus, having *few* diamonds is bad. *at most* + neg = pos, which matches the pos in *win*. Bad result!! Unless we're doing something wrong in the way we're adding these up. Actual negation here probably makes a difference. Should this actually be regarded as a case of neg + *at most* + neg? Overall this would be neg, which would not match the pos in *win*, which could then explain the ✗. For *at least* above then we actually have neg + *at least* + neg = pos which matches the pos in *win* which predicts ✓ but maybe the problem comes from the fact that the pos is derived from two neg's? BTW we previously said that neg+neg is ✓ (from E) but maybe because of multiple negative elements introspective judgments mark it more like "?". Is this what's happening here as well?

Outline

What we have achieved

⋆ Exhaustification with O(nly) relative to scalar and subdomain alternatives yields the empirical patterns w.r.t. scalar implicatures, ignorance, and downward-entailing environments – the polarity sensitivity of SMNs.

⋆ Exhastification wtih E(ven) relative to pre-$O_{ScalAlts}$ 'ed scalar alternatives from a truncated scale yields the valence sensitivity of SMNs.

Open issues

⋆ The PS requirement for SMNs doesn't really follow from anything, it's a stipulation.

⋆ We need to understand better the possible interactions between $O_{ScalAlts}$ and $O_{SubDomAlts}$ (clash between scalar implicatures and ignorance; $O_{ScalAlts}$ can use pre-$O_{SubDomAlts}$'ed alternatives – this gets the reading of SMNs under possibility modals).

⋆ We need to understand better why the valence effect seems to be stronger for SMNs than for CMNs.

⋆ We need to understand better the truncation of the scale that seems to be crucial for deriving the valence data via E(ven).

Outline

(73) $\llbracket \text{three} \rrbracket = 3$; typeshifted: $\llbracket \text{isCard} \rrbracket (3) = \lambda x \,.\, |x| = 3$

(74) $\llbracket \text{Three people quit} \rrbracket$

$$\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$$

$\lambda Q \,.\, \exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ quit

$\exists$ $\lambda x \,.\, |x| = 3 \wedge P(x)$

$\llbracket \text{isCard} \rrbracket (\llbracket \text{three} \rrbracket)$ people
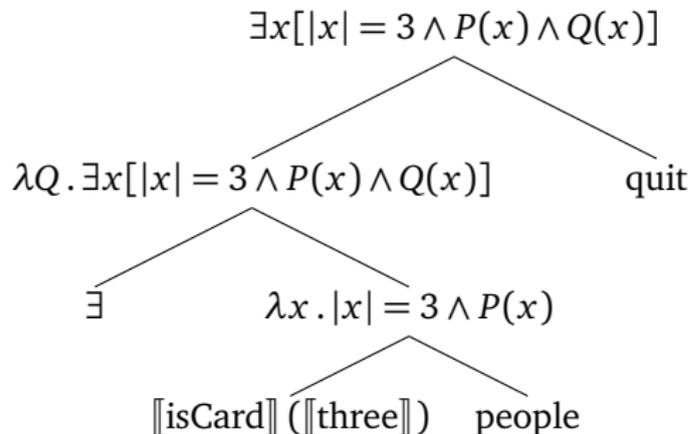
(75) $\llbracket \text{comp} \rrbracket (\llbracket \text{much/little} \rrbracket)(3)$
$= \lambda D_{\langle d,t \rangle} . max(\lambda d . D(d)) \in \overline{\llbracket \text{much/little} \rrbracket (3)}$

(76) More/less than three people quit.

$max(\lambda d . \exists x[|x| = d \land P(x) \land Q(x)]) \in \overline{\llbracket \text{much/little} \rrbracket (3)}$

$\llbracket \text{comp} \rrbracket (\llbracket \text{much/little} \rrbracket)(3) \qquad \lambda d . \exists x[|x| = d \land P(x) \land Q(x)]$

$1 \qquad\qquad \exists x[|x| = d \land P(x) \land Q(x)]$

$\lambda Q . \exists x[|x| = d \land P(x) \land Q(x)] \qquad \text{quit}$

$\exists \qquad \lambda x . |x| = d \land P(x)$

$\llbracket \text{isCard} \rrbracket (t_1) \qquad \text{people}$

(77) $\llbracket \text{at-sup} \rrbracket (\llbracket \text{much/little} \rrbracket)(3)$
$= \lambda D_{\langle d,t \rangle} . max(\lambda d . D(d)) \in \llbracket \text{much/little} \rrbracket (3)$

(78) At most/least three people quit.

$$max(\lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \llbracket \text{much/little} \rrbracket (3)$$

$\llbracket \text{at-sup} \rrbracket (\llbracket \text{much/little} \rrbracket)(3) \qquad \lambda d . \exists x[|x| = d \wedge P(x) \wedge Q(x)]$

$1 \qquad \exists x[|x| = d \wedge P(x) \wedge Q(x)]$

$\lambda Q . \exists x[|x| = d \wedge P(x) \wedge Q(x)] \qquad \text{quit}$

$\exists \qquad \lambda x . |x| = d \wedge P(x)$

$\llbracket \text{isCard} \rrbracket (t_1) \qquad \text{people}$

Positive polarity item patterns
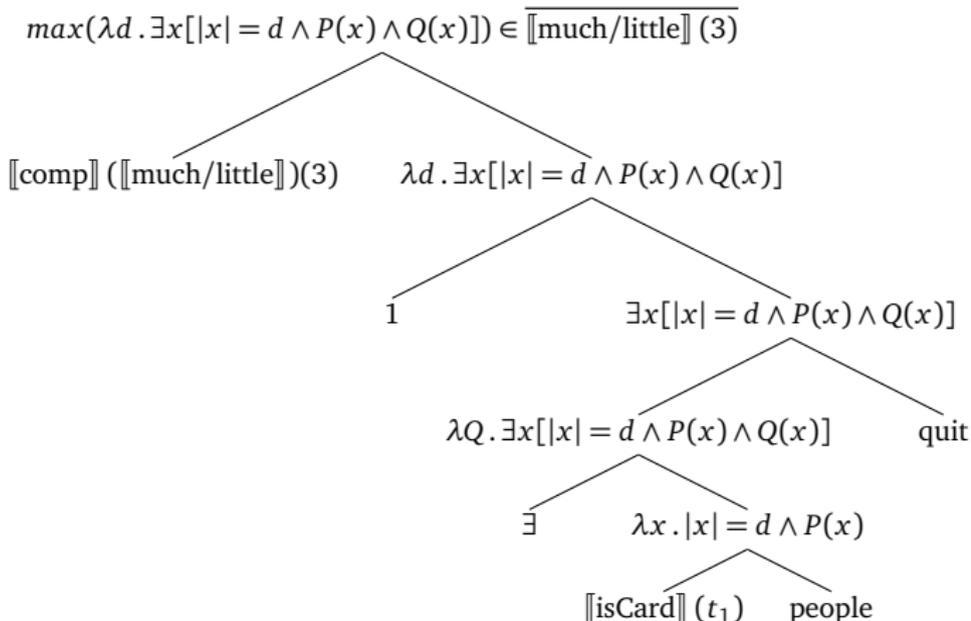
(79) **Antilicensing in the scope of negation**
John didn't see someone.   *not [smn]

(80) **No antilicensing in the antecedent of a conditional / the restriction of a universal**   ✓if/every [smn][ ]
   a. If John saw someone, he should have let us know.
   b. Every student who saw someone should speak up.

(81) **Rescuing in the scope of negation if it is itself embedded in a DE environment**
I doubt that John didn't call someone.   ✓doubt [not [smn]]

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