The alternatives of bare and modified numerals

Introduction. A lot has been written on bare numerals (BNs), comparative-modified numerals (CMs), and superlative-modified numerals (SMs), yet we still lack a unified theory. In light of old and new literature, I revisit four basic phenomena that any such theory needs to address. I observe that none of the existing theories gets all the patterns. I propose a theory that does so, and where any similarities and differences fall out in a principled way from the morphology.

Data. (Entailments) *3 P Q*, *more than 3 P Q*, *at least 3 P Q* entail a lower bound; *less than 3 P Q* and *at most 3 P Q* entail an upper bound.

(1) a. Alice has 3 / more than 3 / at least 3 diamonds, #if not less.
 b. Alice has less than 3 / at most 3 diamonds, #if not more.

(Scalar implicatures) In an unembedded context BNs can give rise to a scalar implicature (*3 P Q* $\rightsquigarrow$ *not 4 P Q*) (Horn 1972), but CMs and SMs don't (Krifka 1999 a.o. since), (2). However, if the granularity of the scale is made coarser, CMs and SMs suddenly do (cf. exp. work by (Cummins et al. 2012), (3). (CMs and SMs also give rise to the predicted (direct or indirect) scalar implicatures in the scope of a universal operator, or in the antecedent of a conditional.)

(2) Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds.
 $\rightsquigarrow \neg$ Alice has 4 / *more than 4 / *less than 2 / *at most 2 / *at least 4 diamonds.

(3) *Context: People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.* (example from (Spector 2014:42))
 John solved more than 5 problems. Peter solved more than 9.
 $\rightsquigarrow \neg$ John solved more than 9.

(Ignorance) In an out-of-the-blue unembedded context, e.g., (2), BNs don't give rise to speaker ignorance inferences, CMs can (cf., e.g., Nouwen 2015, exp. work by Cremers et al. 2017), and SMs must (Geurts and Nouwen 2007 a.o.).

(Acceptability in DE environments) BNs and CMs can take scope below negation but SMs can't (Nilsen 2007, Cohen and Krifka 2014, Spector 2015), (4). All are fine in DE environments such as the antecedent of a conditional or the restrictor of a universal, (5-a)-(5-b).

(4) Alice doesn't have 3 / more than 3 / less than 3 / *at least 3 / *at most 3 diamonds.

(5) a. If Alice has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds, she wins.
 b. Everyone who has 3 / more than 3 / less than 3 / at most 3 / at least 3 diamonds wins.

Previous theories. None of the existing theories (whether semantic or pragmatic) (can) capture all these facts. For example, many are by design such that CMs and SMs *can't* give rise to scalar implicatures, or such that SMs but not CMs can give rise to ignorance (or parallel effects) (in one way or another, all of Krifka 1999, Geurts and Nouwen 2007, Nouwen 2010, Fox and Hackl 2006, Coppock and Brochhagen 2013, Mayr 2013, Kennedy 2015, etc.). Also, in the alternative-based theories the type, number, and nature of the alternatives is usually stipulated.

Proposal. (Truth conditions and presupposition) I propose that in BNs, CMs, and SMs the numeral is just a degree (which in BNs can be typeshifted into a predicate e.g. using Buccola and Spector 2016's `isCard` typeshifter), (6), and *much/little* are positive/negative extent indicators (cf. the extents theory of adjectives in Seuren 1984, Kennedy 1997), (7). BNs have the traditional existential truth conditions in e.g. Krifka (1999), (8). CMs and SMs are defined as in (9)-(10). [sup] carries a presupposition that the domain of $|P \cap Q|$, i.e., the set of degrees denoted by $[\![\text{much/little}]\!]\,(n)$ should contain at least two degrees (paralleling the presupposition of [sup] in adjectives, cf., e.g., Gajewski 2010), (11).

(6) $[\![\text{n}]\!] = n$ (type d) $\qquad [\![\texttt{isCard}]\!]\,(n) = \lambda x\,.\,|x| = n$

(7) $[\![\text{much}]\!]\,(n) = \lambda d\,.\,d \leq n \qquad [\![\text{little}]\!]\,(n) = \lambda d\,.\,d \geq n$

(8) $[\![\text{n P Q}]\!] = (\exists(([\![\texttt{isCard}]\!]\,(n))P))(Q) = 1$ iff $\exists x[|x| = n \wedge P(x) \wedge Q(x)]$

(9) $[\![\text{more/less than n P Q}]\!] = [[\![\text{comp}]\!]\,([\![\text{much/little}]\!])](n)(P)(Q) = 1$ iff $|P \cap Q| \in \overline{[\![\text{much/little}]\!]\,(n)}$

(10) $[\![$at most/least n P Q$]\!] = [[\![\text{sup}]\!]([\![\text{much/little}]\!])](n)(P)(Q) = 1$ iff $|P \cap Q| \in [\![\text{much/little}]\!](n)$
(11) [sup(much/little)](n)(P)(Q) defined iff $|\,[\![\text{much/little}]\!](n)| \geq 2$
(Scalar alternatives) Replace n in the truth conditions of BNs, CMs, and SMs with its scalar alternatives (other numerals, typically from $\mathbb{N}$). **(Subdomain alternatives)** Replace $\overline{[\![\text{much/little}]\!](n)}$ / $[\![\text{much/little}]\!](n)$ in the truth conditions of CMs / SMs with its subsets. Because cf. (11) for SMs $[\![\text{much/little}]\!](n)$ has at least two elements by presupposition, that means that SMs have two subdomain alternatives preloaded, which I take to mean that they must be factored into meaning via implicature. **(Implicature calculation system)** I will adopt Chierchia (2013)'s version of the grammatical theory of implicatures. Specifically, I will use the silent exhaustivity operator O (given a sentence S and a set of alternatives ALT, $O_{\text{ALT}}(S)$ asserts the conjunction of S and the negations of all those members of ALT that can be excluded together without contradiction) and its presuppositional version O^{PS} (same as O, but exhaustification crashes if it does not lead to a properly stronger meaning). I assume that the scalar alternatives of BNs, CMs, and SMs are exhaustified with $O_{ScalAlts}$ and the subdomain alternatives of CMs and SMs with $O^{PS}_{SubDomAlts}$. I also assume that exhaustification parses may include a silent matrix universal epistemic modal $\boxdot$ (cf. Kratzer and Shimoyama 2002, Chierchia 2013's silent last resort $\Box$ / (neo-)Gricean Bel_S / Meyer 2013's K).
Results. (Entailments) Straightforwardly captured, as the reader can verify. **(Scalar implicatures)** $O_{ScalAlts}$ yields all the attested (direct or indirect) scalar implicatures (essentially the same results as the original Gricean story, plus additional desirable ones, i.e., the embedded scalar implicatures for BNs discussed by Kennedy 2015). The unattested ones leading to 'exactly *n*' meanings for CMs and SMs in fact never arise – $O_{ScalAlts}$ clashes with $O^{PS}_{SubDomAlts}$ (see Ignorance below) and the clash can only be resolved by pruning the offending scalar implicature (which is essentially what coarse granularity contexts do by default). **(Ignorance, acceptability in DE environments)** (Reasoning here parallel to, e.g., Kennedy 2015's neo-Gricean implicature account of Ignorance for SMs and to Spector 2015's grammatical implicature account of Ignorance and infelicity under negation for SMs.) I derive these patterns from the subdomain alternatives of CMs and SMs. To simplify our discussion, observe that CMs and SMs are of the form $|P \cap Q| \in D$, and their SubDomAlts are of the form $|P \cap Q| \in A$, where $A \subseteq D$. Also recall that the subdomain alternatives of SMs must be used, so SMs can only have parses with $O^{PS}_{SubDomAlts}$, and are bad if $O^{PS}_{SubDomAlts}$ crashes. Now consider the potential positive and negative LFs of CMs and SMs in (12)-(13). In the positive case there are two coherent parses: (12-a) with no $O^{PS}_{SubDomAlts}$ and (12-c) yielding ignorance. CMs can have either one of them (optional ignorance) but SMs are restricted to the $O^{PS}_{SubDomAlts}$ one (obligatory ignorance). In the negative case there is just one coherent parse, the (12-a) parse with no $O^{PS}_{SubDomAlts}$. This parse is open to CMs (acceptable under negation) but not to SMs (bad under negation). The felicity of SMs in presuppositional DE environments such as (5-a) and (5-b) can also be derived if we consider the effect of O^{PS} on the presupposition (Chierchia 2013, Spector and Sudo 2017, Nicolae 2017).

(12)
a. $|P \cap Q| \in D$
b. $*O^{PS}_{SubDomAlts}(|P \cap Q| \in D)$
c. $O^{PS}_{SubDomAlts} \boxdot (|P \cap Q| \in D)$
$(\forall A \subset D{:}) \rightsquigarrow \neg\boxdot(|P \cap Q| \in A)$

(13)
a. $\neg(|P \cap Q| \in D)$
b. $*\neg O^{PS}_{SubDomAlts}(|P \cap Q| \in D)$
c. $*O^{PS}_{SubDomAlts}(\neg(|P \cap Q| \in D))$
d. $*O^{PS}_{SubDomAlts}(\boxdot\neg(|P \cap Q| \in D))$

Conclusion. I provide a unified implicature account of bare and modified numerals where their similarities and differences fall out in a principled way from their morphological parts. The theory covers more ground than existing literature, and with fewer stipulations.

Selected references. Chierchia (2013). Logic in grammar. Cummins et al. (2012). Granularity and scalar implicature in numerical expressions. Gajewski (2010). Superlatives, NPIs, and *most*. Kennedy (1997). Projecting the adjective. Nicolae, A. C. (2017). Deriving the positive polarity behavior of plain disjunction. Spector (2015). Why are class B modifiers global PPIs?