

The alternatives of bare and modified numerals

3 (BNS)	(ScalAlts)
more/less than 3 (CMNs)	(ScalAlts), (SubDomAlts)
at most/least 3 (SMNs)	(ScalAlts), SubDomAlts

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Preview

- ★ *3, more/less than 3, and at least/most 3* differ w.r.t. (at least)
 - entailments,
 - scalar implicatures,
 - ignorance, and
 - acceptability in downward-entailing environments.
- ★ Many theories have been proposed to capture these differences.
- ★ Lately a move towards alternative-based theories.
- ★ Promising results, but also empirical and conceptual issues.
- ★ I will propose a theory that overcomes these issues.

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★ *3 / more than 3 / at least 3* carry lower-bounding entailments.

- (1) a. Alice has 3 diamonds.
b. \rightsquigarrow not 2 or less
c. Alice has 3 diamonds, # if not less.

★ *less than 3 / at most 3* carry upper-bounding entailments.

- (2) a. Alice has less than 3 diamonds.
b. \rightsquigarrow not 3 or more
c. Alice has less than 3 diamonds, #if not more.

★ Existing proposals: Multiple possible solutions, typically not compositional down to the smallest pieces.

★ **We want** one that gets these **entailments** with ease and also minimally uncovers **the uniform contribution of the numeral, *much/little*, or [-er]/[at -est]** in producing these entailments.

Scalar implicatures I

- ★ BNs also carry upper-bounding scalar implicatures. [Horn, 1972]

- (3) a. Alice has 3 diamonds.
b. \rightsquigarrow not 4 yields 'exactly 3' meaning ✓
c. Alice has 3 diamonds, if not 4.

- ★ CMNs and SMNs don't seem to. [Krifka, 1999]

- (4) a. Alice has more than 3 diamonds.
b. \nrightarrow not more than 4 yields 'exactly 4' meaning ✗

- ★ Existing proposals: No scalar implicatures for CMNs and SMNs.

Scalar implicatures II

- ★ But in certain contexts all give rise to scalar implicatures!

- (5) a. If you have at least 3 diamonds, you win.
b. \rightsquigarrow not if at least 2

- ★ And in some none do:

- (6) a. Alice doesn't have 3 diamonds.
b. $\cancel{\rightsquigarrow}$ not not 2 yields 'exactly 2' meaning ✗

- ★ We want scalar implicatures for all!
- ★ We need a separate mechanism to rule out certain implicatures.

Scalar implicatures III

★ With coarser granularity, CMNs and SMNs can give rise scalar implicatures too. [\[Spector, 2014, Cummins et al., 2012, Enguehard, 2018\]](#)

(7) *Grades are given based on the number of problems solved. People who solve more than 5 problems but fewer than 9 problems get a B, and people who solve 9 problems or more get an A.*

a. John solved more than 5 problems.

b. \rightsquigarrow not more than 9 (he gets a B) example from [\[Spector, 2014\]](#)

★ That is true of BNs in the problem cases also.

(8) a. Alice doesn't have 3 diamonds.

b. \nrightarrow not not 1 (she does have some)

Ignorance I

- ★ SMNs give rise to strong speaker ignorance inferences.

(9) I have 3 / more than 2 / ??at least 3 children.

- ★ Existing proposals: e.g., [Büring, 2008, Kennedy, 2015, Spector, 2015]
 - SMNs are underlyingly disjunctive (*at least 3* = exactly 3 or more than 3) and have domain alternatives (the individual disjuncts).
 - Ignorance inferences are implicatures from these alternatives.
 - Nothing of this sort is assumed / derived for CMNs.

Ignorance II

- ★ CMNs give rise to ignorance inferences too! [Cremers et al., 2017]

(10) [A:] How many diamonds does Alice have? [B:] More than 3.

- ★ Unlike BNs and like SMNs, CMNs are compatible with ignorance:

(11) I don't know how many diamonds Alice has, but she has # 3 / more than 3 / at least 3.

- ★ Unlike CMNs, SMNs are incompatible with exact knowledge.

[Nouwen, 2015]

(12) There were exactly 62 mistakes in the manuscript, so that's more than 50 / # at least 50.

- ★ We want ignorance implicatures for CMNs too!
- ★ We want ignorance to be weaker for CMNs than for SMNs.

Acceptability in DE environments I

- ★ SMNs are bad under negation.

[Nilsen, 2007, Geurts and Nouwen, 2007, Cohen and Krifka, 2014, Spector, 2015]

(13) Alice doesn't have *at least three / *at most three diamonds.

→ Alice has 2 or less / 4 or more diamonds.

✗

- ★ Existing proposals: The domain alternatives of SMNs are obligatory and must lead to a stronger meaning, but that cannot happen in a DE environment like negation.

[Spector, 2015]

Acceptability in DE environments II

- ★ SMNs are okay in the antecedent of a conditional or the restriction of a universal!

[Geurts and Nouwen, 2007, Cohen and Krifka, 2014, Spector, 2015]

(14) If Alice has at least 3 diamonds, she wins.

(15) Everyone who has at least 3 diamonds wins.

- ★ We want a solution that can distinguish between various types of DE environments!

Summary and preview of proposal

- ★ BNs, CMNs, and SMNs are non-uniform w.r.t.
 - Entailments
 - Scalar implicatures
 - Ignorance
 - Acceptability in DE environments
- ★ The existing alternative-based proposals are promising, but still:
 - they take into evidence an incomplete dataset;
 - they make non-uniform stipulations about the alternatives;
 - they fail to capture all the patterns we saw.
- ★ In this talk:
 - I take into evidence a revised and extended dataset;
 - I derive the alternatives of BNs, CMNs, and SMNs in a uniform way from their truth conditions;
 - I show how, with certain general assumptions about implicature calculation, we get all the patterns we saw.

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Proposal: Truth conditions and presupposition

the numeral

|| [Link, 1983, Buccola and Spector, 2016]

$$\llbracket n \rrbracket = n$$

$$\llbracket \text{is}_{\text{Card}} \rrbracket (n) = \lambda x_e . |x| = n$$

much/little

|| [Seuren, 1984, Kennedy, 1997]

$$\llbracket \text{much} \rrbracket (n) = \lambda d . d \leq n$$

$$\llbracket \text{little} \rrbracket (n) = \lambda d . d \geq n$$

truth conditions ^t

|| [Krifka, 1999, Von Stechow, 2005, Heim, 2007, Hackl, 2009]

$$(\exists (n P))(Q) = 1 \text{ iff } \exists x[|x| = n \wedge P(x) \wedge Q(x)]$$

$$[\text{comp}](\text{much/little})(n)(P)(Q) = 1 \text{ iff } |P \cap Q| \in \overline{\llbracket \text{much/little} \rrbracket (n)}$$

$$[\text{at-sup}](\text{much/little})(n)(P)(Q) = 1 \text{ iff } |P \cap Q| \in \llbracket \text{much/little} \rrbracket (n)$$

the presupposition of at-sup

|| [Hackl, 2009, Gajewski, 2010]

$$| \llbracket \text{much/little} \rrbracket (n) | \geq 2$$

✓ Entailments

(16) $3 P Q$:

$$\exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \quad (\text{l.b.})$$

(17) *more than 3* $P Q$:

$$|P \cap Q| \in \overline{\llbracket \text{much} \rrbracket} (3) \Leftrightarrow |P \cap Q| \in \{4, 5, \dots\} \quad (\text{l.b.})$$

(18) *less than 3* $P Q$:

$$|P \cap Q| \in \llbracket \text{little} \rrbracket (3) \Leftrightarrow |P \cap Q| \in \{\dots, 0, 1, 2\} \quad (\text{u.b.})$$

(19) *at most 3* $P Q$:

$$|P \cap Q| \in \llbracket \text{much} \rrbracket (3) \Leftrightarrow |P \cap Q| \in \{\dots, 0, 1, 2, 3\} \quad (\text{u.b.})$$

(20) *at least 3* $P Q$:

$$|P \cap Q| \in \llbracket \text{little} \rrbracket (3) \Leftrightarrow |P \cap Q| \in \{3, 4, \dots\} \quad (\text{l.b.})$$

Proposal: Alternatives

Scalar alternatives: Replace the n -domain with an m -domain.

BNs: $\{\exists x[|x| = m \wedge P(x) \wedge Q(x)] : m \in S\}$

CMs: $\{|P \cap Q| \in \overline{\llbracket \text{much/little} \rrbracket (m)}} : m \in S\}$

SMs: $\{|P \cap Q| \in \llbracket \text{much/little} \rrbracket (m)} : m \in S\}$

Subdomain alternatives: Replace the n -domain with its subsets.

BNs: NA (the numeral argument is just a degree)

CMs: $\{|P \cap Q| \in A : A \subseteq \overline{\llbracket \text{much/little} \rrbracket (n)}}\}$

SMs: $\{|P \cap Q| \in A : A \subseteq \llbracket \text{much/little} \rrbracket (n)}\}$

active by presup!

obligatory exhaustification relative to SubDomAlts

Examples

(21) BNs: $3 P Q$

- Truth conditions: $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$
- ScalAlts: $\{\dots, \exists x[|x| = 2\dots], \exists x[|x| = 4\dots, \dots]\}$
- SubDomAlts: NA

(22) CMNs: e.g., *more than 3 P Q*

- Truth conditions: $|P \cap Q| \in \overline{\llbracket \text{much} \rrbracket} (3)$
- ScalAlts: $\{\dots, |P \cap Q| \in \overline{\llbracket \text{much} \rrbracket} (2), |P \cap Q| \in \overline{\llbracket \text{much} \rrbracket} (4), \dots\}$
- SubDomAlts: $\{|P \cap Q| \in A : A \subseteq \overline{\llbracket \text{much} \rrbracket} (3)\}$

(23) SMNs: e.g., *at least 3 P Q*

- Truth conditions: $|P \cap Q| \in \llbracket \text{little} \rrbracket (3)$
- ScalAlts: $\{\dots, |P \cap Q| \in \llbracket \text{little} \rrbracket (2), |P \cap Q| \in \llbracket \text{little} \rrbracket (4), \dots\}$
- SubDomAlts: $\{|P \cap Q| \in A : A \subseteq \llbracket \text{little} \rrbracket (3)\}$

active!

O to exhaustify the scalar alternatives of BNs, CMNs, and SMNs

$$(24) \llbracket O_{ALT}(p) \rrbracket = p \wedge \forall q \in ALT [q \rightarrow p \subseteq q]$$

O^{PS} to exhaustify the subdomain alternatives of CMNs and SMNs

- ★ A version of O that
 - takes into account presuppositions:

$$(25) \llbracket O_{ALT}^S(p) \rrbracket = \pi(p) \wedge \forall q \in ALT [\pi(q) \rightarrow \pi(p) \subseteq \pi(q)],$$

- requires a properly stronger result:

$$(26) \llbracket O_{ALT}^{PS}(p) \rrbracket \text{ is defined iff } O_{ALT}^S(p) \subset p.$$

Whenever defined, $\llbracket O_{ALT}^{PS}(p) \rrbracket = \llbracket O_{ALT}^S(p) \rrbracket$.

□ last resort, silent, matrix-level, universal doxastic modal



(27) Alice has 3 diamonds.

$$\begin{aligned} \text{a. } O_{\text{ScalAlts}} & (\exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \wedge) \\ & = \exists x[|x| = 3 \wedge P(x) \wedge Q(x)] \wedge \\ & \neg \exists x[|x| = 4 \wedge P(x) \wedge Q(x)] \wedge \dots \end{aligned}$$

not 4 ✓

(28) Alice has at least 3 diamonds.

$$\begin{aligned} \text{a. } O_{\text{ScalAlts}} & (|P \cap Q| \in \llbracket \text{little} \rrbracket (3)) \\ & = |P \cap Q| \in \llbracket \text{little} \rrbracket (3) \wedge \\ & \neg |P \cap Q| \in \llbracket \text{little} \rrbracket (5) \wedge \dots \end{aligned}$$

not at least 5 ✓

(29) If Alice has more than 3 diamonds, she wins.

$$\begin{aligned} \text{a. } O_{\text{ScalAlts}} & (\overline{(|P \cap Q| \in \llbracket \text{much} \rrbracket (3))} \rightarrow \text{win}) \\ & = \overline{(|P \cap Q| \in \llbracket \text{much} \rrbracket (3))} \rightarrow \text{win} \wedge \\ & \neg \overline{(|P \cap Q| \in \llbracket \text{much} \rrbracket (2))} \rightarrow \text{win} \wedge \dots \end{aligned}$$

not if more than 2 ✓

- ★ And so on. We can derive all the attested scalar implicatures.
- ★ Scalar implicatures may be restricted by granularity.
- ★ In unembedded contexts this effect is compounded by ignorance. 19 / 38

Implicatures from SubDomAlts: Ignorance



(30) Alice has more/less than 3 / at most/least 3 diamonds.

a. $O_{SubDomAlts}^{PS} (|P \cap Q| \in D)$

$$= |P \cap Q| \in D \wedge$$

$$\neg |P \cap Q| \in A \wedge$$

$$\neg |P \cap Q| \in B \wedge \dots, \text{ for all } A, B, \dots \subset D, = \perp$$

contradiction ✗

b. $O_{SubDomAlts}^{PS} (\Box |P \cap Q| \in D)$

$$= \Box |P \cap Q| \in D \wedge$$

$$\neg \Box |P \cap Q| \in A \wedge$$

$$\neg \Box |P \cap Q| \in B \wedge \dots, \text{ for all } A, B, \dots \subset D$$

ignorance ✓

- ★ The only consistent $O_{SubDomAlts}^{PS}$ parse yields ignorance.
- ★ SMNs can only have an $O_{SubDomAlts}^{PS}$ parse, so *(ignorance)
- ★ CMNs can also have a parse without $O_{SubDomAlts}^{PS}$, so (ignorance).

Scalar implicatures vs. ignorance implicatures

(31) Alice has more than 2 / at least 3 diamonds.

$$O_{SubDomAlts}^{PS} \sqsupset O_{ScalAlts} (|P \cap Q| \in \{3, 4, \dots\})$$

$$= \sqsupset O_{ScalAlts} (|P \cap Q| \in \{3, 4, \dots\}) \wedge$$

$$\neg \sqsupset (|P \cap Q| \in \{3\}) \wedge$$

$$\neg \sqsupset (|P \cap Q| \in \{4, 7\}) \wedge \dots$$

$$= \sqsupset (|P \cap Q| \in \{3, 4, \dots\} \wedge \neg |P \cap Q| \in \{4, \dots\}) \wedge$$

$$\neg \sqsupset (|P \cap Q| \in \{3\}) \wedge$$

$$\neg \sqsupset (|P \cap Q| \in \{4, 7\}) \wedge \dots$$

$$= \underline{\sqsupset (|P \cap Q| \in \{3\}) \wedge \neg \sqsupset (|P \cap Q| \in \{3\})} \wedge$$

$$\neg \sqsupset (|P \cap Q| \in \{4, 7\}) \wedge \dots = \perp$$

contradiction ✗

★ Prune offending SubDomAlts? That would violate $O_{SubDomAlts}^{PS}$ ✗

★ Prune offending ScalAlt? ✓



(32) Alice doesn't have more/less than three / *at most/least three diamonds.

a. $\neg O_{SubDomAlts}^{PS} (|P \cap Q| \in D)$

contradiction ✗

b. $O_{SubDomAlts}^{PS} (\neg|P \cap Q| \in D)$

PS violated ✗

c. $O_{SubDomAlts}^{PS} (\Box \neg|P \cap Q| \in D)$

PS violated ✗

- ★ All $O_{SubDomAlts}^{PS}$ parses fail.
- ★ SMNs cannot have a non- $O_{SubDomAlts}^{PS}$ parse, so bad.
- ★ CMNs can be parsed without $O_{SubDomAlts}^{PS}$, so okay.



(33) $O_{SubDomAlts}^{PS}$ (Everyone who has at least 3 diamonds wins.)

Prejacent: $\forall x[\# \text{ di } x \text{ has } \in D \rightarrow \dots] \wedge \exists x[\# \text{ of di } x \text{ has } \in D]$



SubDomAlt: $\forall x[\# \text{ di } x \text{ has } \in D' \rightarrow \dots] \wedge \exists x[\# \text{ of di } x \text{ has } \in D']$

- ★ SubDomAlts not entailed, so they must be false.
- ★ However, negating them leads to contradiction.
- ★ We can rescue the parse with \Box :

(34) $\Box \exists x[\# \text{ of di } x \text{ has } \in D] \wedge \neg \Box \exists x[\# \text{ of di } x \text{ has } \in D']$

PS satisfied ✓

- ★ Thus there is a consistent $O_{SubDomAlts}^{PS}$ parse for SMNs, which is why they are felicitous in this type of DE environments.

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(35) LeBron scored at most 20 points (and it's even possible that he didn't score any points at all).

$$O_{ScalAlts} (|P \cap Q| \in \llbracket \text{much} \rrbracket (20))$$
$$= |P \cap Q| \in \llbracket \text{much} \rrbracket (20) \wedge$$
$$\neg |P \cap Q| \in \llbracket \text{much} \rrbracket (18) \wedge$$
$$\neg |P \cap Q| \in \llbracket \text{much} \rrbracket (17) \wedge$$
$$\dots$$
$$\neg |P \cap Q| \in \llbracket \text{much} \rrbracket (0)$$

existential implicature ✓

- ★ Lower-bounding inference is a scalar implicature, which is why it is defeasible.
- ★ The same can be observed for *less than*.
- ★ Both follow if we assume CMNs and SMNs have scalar alternatives.

The ‘not possible more’ reading of *at most* under \diamond

(36) a. You are allowed to drink at most one beer.

b. $O_{ScalAlts} (O_{SubDomAlts}^{PS} (\diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1)))$

c. Prejacent: $O_{ExhSubDomAlts}^{PS} (\diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1))$

d. ScalAlts: $O_{ExhSubDomAlts}^{PS} (\diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (m))$

e. Outcome:

$O_{ScalAlts} (O_{ExhSubDomAlts}^{PS} (\diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1)))$

$= O_{ExhSubDomAlts}^{PS} (\diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (1)) \wedge$

$\neg O_{SubDomAlts}^{PS} (\diamond |P \cap Q| \in \llbracket \text{much} \rrbracket (2))$

$= \diamond |P \cap Q| \in \{0\} \wedge \diamond |P \cap Q| \in \{1\} \wedge$

$\neg(\diamond |P \cap Q| \in \{0\} \wedge \diamond |P \cap Q| \in \{1\} \wedge \diamond |P \cap Q| \in \{2\})$

not possible more ✓

★ This follows from a system where $O_{SubDomAlts}$ can apply to pre-exhaustified alternatives, where $O_{SubDomAlts}$ and $O_{ScalAlts}$ can be manipulated separately, and where $O_{SubDomAlts}$ can be part of the prejacent and the alternatives operated on by $O_{ScalAlts}$.

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Conclusion

- ★ A unified account of bare, comparative-modified, and superlative-modified numerals that
 - captures more patterns than previous accounts; and
 - derives them from
 - truth conditions and alternatives obtained in a uniform way from the morphological pieces of BNs, CMNs, and SMNs, and
 - general implicature calculation mechanisms, using general recipes for deriving scalar implicatures, ignorance effects, polarity sensitivity, or free choice behavior.

Open issues

★ How does superlativity in SMNs (at-sup) connect to superlativity in adjectives (sup)?

★ Why Proper Strengthening?

(At present it is a stipulation. We could replace it with a ban on vacuous exhaustification but I think in the general case that might be too strong. It however seems to be a necessary general assumption for items with a positive polarity behavior such as SMNs - [Spector, 2014, Nicolae, 2017]. Parametric choice? Is there any evidence of SMNs that are not PPIs?)

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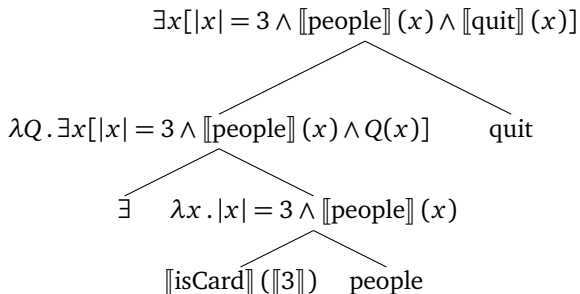
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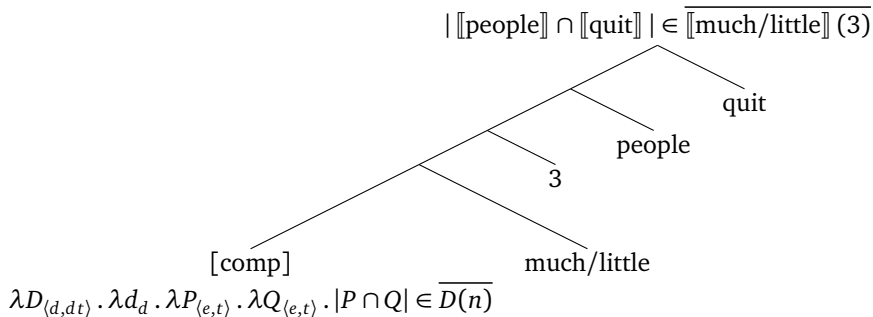
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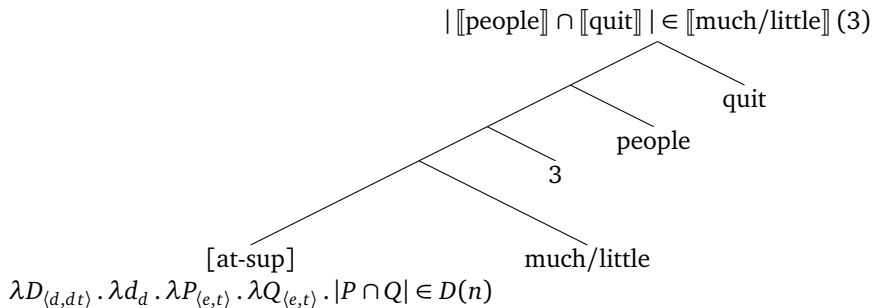
(37) 3 people quit.



(38) More/less than 3 people quit.



(39) At most/least 3 people quit.



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