Ignorance and polarity sensitivity in modified numerals and beyond

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Overview. I investigate ignorance and polarity sensitivity in items like *some*, comparative-modified numerals (CMNs), and superlative-modified numerals (SMNs). Previous literature shows that this type of phenomena can be investigated fruitfully in an alternatives-and-exhaustification framework. Building on this, I sketch an account for *some* and then extend it to CMNs and SMNs, deriving any similarities and differences from their domain alternatives (which here are themselves derived in a principled way from a novel decomposition of modified numerals into their morphological pieces).

***Some* – data and analysis.** *Some* in (1) triggers a strong speaker ignorance effect – the speaker doesn't know which student. *Some* also exhibits polarity sensitivity, being degraded in the scope of negation, (2).

(1) *Some student was looking for you. Namely, John. (obligatory ignorance)

(2) *I didn't talk to some student. (anti-negativity)

Both these types of patterns can be captured within an alternatives-and-exhaustification framework. For concreteness, let's adopt Chierchia (2013)'s (1) contradiction-based version of the grammatical theory of implicatures, which relies on a silent exhaustivity operator O (akin to a silent *only*) defined such that, given a sentence S ('prejacent') and a set of alternatives ALT, $O_{\text{ALT}}(S)$ asserts the conjunction of S and the negations of all the alternatives from ALT that are not entailed by S; and (2) way of handling empirical variation relative to ignorance and polarity sensitivity, which relies on parametrized lexical specifications regarding the types of alternatives that an item may activate and the mode of exhaustification that it selects for those alternatives. Suppose now that *some*, whose truth conditions make reference to a domain of individuals ($\exists x \in D[P(x) \wedge Q(x)]$), obligatorily activates domain alternatives (DAs) (obtained by replacing D with any $D' \subset D$) and moreover requires that exhaustification relative to these alternatives must lead to strengthening (the Proper Strengthening requirement, indicated as a superscript on 'O': O^{PS}). These assumptions derive all the patterns above. Below I illustrate this for a domain with just two individuals, D = {a, b}, abbreviating $\exists x \in \{a, b\}[P(x) \wedge Q(x)]$ as $\{a, b\}$. (3) shows the unembedded case: The DAs are stronger than the prejacent, so we have to negate them, but this leads to contradiction, (3a). One way to rescue the parse is to insert a silent, matrix-level, universal epistemic modal (which I will write as $\boxdot$ and conceptualize as in Kratzer and Shimoyama 2002 as a last resort mechanism; akin to the (neo-)Gricean Bel_S / Meyer 2013's K) in between the exhaustivity operator and its prejacent; this produces ignorance, (3b). Since O^{PS}_{DA} is obligatory and this is the only consistent O^{PS}_{DA} parse for the unembedded case, the result is *obligatory* ignorance. (4) shows the case of embedding under negation: Exhaustifying below negation yields a contradiction, (4a). Exhaustifying above negation (with or without $\boxdot$) violates proper strengthening – the DAs are already entailed, so exhaustification is vacuous, (4b)-(4c). Thus there is no good O^{PS}_{DA} parse for the negative case, so we derive anti-negativity.

(3) a. $O^{PS}_{DA}(\{a, b\}) = \{a, b\} \wedge \neg\{a\} \wedge \neg\{b\} = \bot$ (G(rammatically)-trivial) ✗

b. $O^{PS}_{DA}(\boxdot\{a, b\}) = \boxdot\{a, b\} \wedge \neg \boxdot \{a\} \wedge \neg \boxdot \{b\}$ (ignorance implicatures) ✓

(4) a. $\neg O^{PS}_{DA}(\{a, b\}) = \neg(\{a, b\} \wedge \neg\{a\} \wedge \neg\{b\}) = \neg(\bot) = \top$ (G(rammatically)-trivial) ✗

b. $O^{PS}_{DA}(\neg\{a, b\}) = \neg\{a, b\}$ (the DAs are entailed, so O^{PS}_{DA} is vacuous, which violates PS) ✗

c. $O^{PS}_{DA}(\boxdot\neg\{a, b\}) = \boxdot\neg\{a, b\}$ (the DAs are entailed, so O^{PS}_{DA} is vacuous, which violates PS) ✗

Modified numerals – data. We find ignorance and polarity sensitivity effects in modified numerals also, and they are as follows: Both CMNs and SMNs are compatible with an ignorant speaker (Nouwen 2015, Cremers et al. 2017), (5a), but for SMNs this goes beyond compatibility – it is a requirement (Geurts and Nouwen 2007 a.o.), (5b). And CMNs can take scope below negation but SMNs can't (Nilsen 2007, Cohen and Krifka 2014, Spector 2015, Mihoc and Davidson 2017, a.o.), (6).

(5) a. I don't know how many windows this house has, but it's less than 11 / at most 10.

b. Look, this house has 9 windows. Therefore it has less than 11 / *at most 10 windows.

(6) This house doesn't have less than 11 / *at most 10 windows.

To sum up, SMNs are empirically just like *some* – they exhibit obligatory ignorance and anti-negativity. On the other hand, despite their surface morphological similarity to SMNs, CMNs are different: they are compatible with both ignorance and certainty, and they do not exhibit anti-negativity.

Modified numerals – analysis. The solution for *some* crucially relied on assumptions about its DAs. But what are the DAs in the case of CMNs and SMNs? I propose that they follow naturally from their truth conditions. **(The truth conditions of CMNs and SMNs)** The truth conditions of *more/less than n P Q* and *at least/most n P Q* are obtained from a numeral n, which I analyze as a degree (that, if needed, can be typeshifted into a predicate; cf. Buccola and Spector 2016); *much/little*, which I analyze as extent indicators (cf. the extent theory of adjectives in Seuren 1984, Kennedy 1997), (8); and [comp]/[at-sup], which I analyze as functions that take in ⟦much/little⟧, n, P, and Q, and yield true iff $|P \cap Q|$ is a number in the set of degrees given by the complement of the positive/negative extent of n (CMNS), (9) / the positive/negative extent of n (SMNs), (10).

(7) ⟦3⟧ = 3 (type d)

(8) ⟦much⟧ $(3) = \lambda d\,.\,d \leq 3$ $\overline{⟦\text{little}⟧\ (3) = \lambda d\,.\,d \geq 3}$

(9) ⟦more/less than 3 P Q⟧ = [⟦comp⟧ (⟦much/little⟧)]$(3)(P)(Q) = 1$ iff $|P \cap Q| \in \overline{⟦\text{much/little}⟧\ (3)}$

(10) ⟦at least/most 3 P Q⟧ = [⟦at-sup⟧ (⟦little/much⟧)]$(3)(P)(Q) = 1$ iff $|P \cap Q| \in$ ⟦little/much⟧ (3)

(The shape of the domain alternatives of CMNs and SMNs) The DAs are obtained by replacing the set of degrees $\overline{⟦\text{much/little}⟧\ (n)}$ / ⟦much/little⟧ (n) in (9), (10) (the domain of $|P \cap Q|$) with its subsets.

(Ignorance and polarity sensitivity: SMNs) SMNs are just like *some* – they exhibit obligatory ignorance and anti-negativity. Just as for *some*, (3)-(4), we can get this from obligatory DAs and proper strengthening. I illustrate this for *at most 2 P Q*, abbreviating, e.g., $|P \cap Q| \in \{0,1,2\}$ as $\{0,1,2\}$.

(11) $O^{PS}_{DA}\ (\boxdot\{0,1,2\}) = \boxdot\{0,1,2\} \wedge \neg\boxdot\{0,1\} \wedge \neg\boxdot\{0,2\} \wedge \neg\boxdot\{1,2\} \wedge \neg\boxdot\{0\} \wedge \neg\boxdot\{1\} \wedge \neg\boxdot\{2\}$ ✓

(12) $O^{PS}_{DA}\ (\neg\{0,1,2\}) = \neg\{0,1,2\}$ ✗

(Ignorance and polarity sensitivity: CMNs) Unlike *some* or SMNs, CMNs are compatible with both ignorance *and certainty*. I propose this follows from the following: (1) *some*, SMNs, and CMNs all admit of pre-exhaustified (Exh-DAs) (an assumption that is independently motivated, as I will show) and (2) unlike *some* or SMNs, CMNs can ignore their singleton set DAs (paralleling assumptions used to derive variation among epistemic indefinites in, e.g., Alonso-Ovalle and Menéndez-Benito 2010). For *less than 3 P Q* we could thus have (13) – a parse compatible with either total ignorance or with certainty.

(13) $O_{Exh\text{-}DA}\ (\boxdot\{0,1,2\}) = \boxdot\{0,1,2\} \wedge \neg O_{DA}\ (\boxdot\{0,1\}) \wedge \neg O_{DA}\ (\boxdot\{0,2\}) \wedge \neg O_{DA}\ (\boxdot\{0,1,2\})$

$= \boxdot\{0,1,2\} \wedge (\boxdot\{0,1\} \rightarrow \boxdot\{0\} \vee \boxdot\{1\}) \wedge (\boxdot\{0,2\} \rightarrow \boxdot\{0\} \vee \boxdot\{2\}) \wedge (\boxdot\{1,2\} \rightarrow \boxdot\{1\} \vee \boxdot\{2\})$

Also, unlike *some* or SMNs, CMNs are fine under negation. This follows if we assume that they do not have the proper strengthening requirement (as argued for, e.g., English *any* or German *irgendein*).

Other results. (1) The fact that *some* or SMNs are okay in the restriction of conditionals/universals can also be derived – the existential presupposition of these environments helps satisfy the proper strengthening requirement. (2) The truth conditions above also naturally yield scalar alternatives for CMNs and SMNs; as I will show, these interact in interesting ways with the pieces we manipulated above.

Conclusion and open issues. I provide an account of ignorance and polarity sensitivity that captures both the similarity of these phenomena in otherwise unrelated items (*some*, SMNs) but also minimal contrasts within items that are otherwise very similar (CMNs, SMNs). Open questions: Is there a language where SMNs do not come with the proper strengthening requirement, or one where CMNs do?

References (selected)

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