Ignorance and polarity sensitivity in modified numerals and beyond

Teodora Mihoc
Department of Linguistics, Harvard University

Puzzle

Or, *some* NP_{SG}, comparative-modified numerals (CMNs; *more/less than 3*), and superlative-modified numerals (SMNs; *at least/most 3*):

all can give rise to ignorance

(1) $(\Diamond a/m \wedge \Diamond b/n \wedge \ldots)$
- a. Toby or Sue cheated.
- b. Some student cheated.
- c. More than 2 students cheated.
- d. At least 3 students cheated.

they differ w.r.t. the strength of ignorance

(2) $(\Box a/n(\wedge \neg\Box b/m \wedge \neg\Box \ldots))$
- a. Toby cheated.
  #Therefore, Toby or Sue cheated.
- b. Toby cheated.
  Therefore, some student in your class cheated.
- c. 3 students cheated.
  Therefore, more than 2 students cheated.
- d. 3 students cheated.
  #Therefore, at least 3 students cheated. [1]

they differ w.r.t. anti-negativity

(3) (not > [item])
- a. John didn't call Toby or Sue.
- b.#John didn't call some student. [2]
- c. John didn't call more than 2 students.
- d.#John didn't call at least 3 students. [3]

(4) (every > [item])
- a. Everyone who called Toby or Sue won.
- b. Everyone who called some student won.
- c. Everyone who called more than 2 students won.
- d. Everyone who called at least 3 students won. [3]

		ignorance	
		total	partial
anti-negativity	no	*or*	CMNs
	yes	SMNs	*some* NP_{SG}

Table: Ignorance and anti-negativity.

What can explain these patterns?

A unified approach to ignorance and anti-negativity

Basic ideas

⋆ Silent exhaustivity operator: $[\![\mathrm{O}_{\mathrm{ALT}}\ p]\!]^{g,w} = [\![\mathrm{p}]\!]^{g,w} \wedge \forall q \in [\![\mathrm{p}]\!]^{\mathrm{ALT}} [[\![\mathrm{q}]\!]^{g,w} \rightarrow \lambda w' .\ [\![\mathrm{p}]\!]^{g,w'} \subseteq q]$. [4]
⋆ Ignorance is a Free Choice (FC) effect. FC comes out of recursive exhaustification or exhaustification plus Innocent Inclusion or exhaustification relative to pre-exhaustified subdomain alternatives. I adopt the latter, $\mathrm{O}_{\text{Exh-DA}}$. [5, 6, 4]
⋆ Partial FC effects: Obligatory $\mathrm{O}_{\text{Exh-DA}}$ + tolerance to exhaustification relative to just a (natural) subset of the set of subdomain alternatives (e.g., just non-singletons, $\mathrm{O}_{\text{Exh-NonSg-DA}}$, or just singletons, $\mathrm{O}_{\text{Exh-Sg-DA}}$). [4]
⋆ Anti-negativity: Obligatory $\mathrm{O}_{\text{Exh-DA}}$ + a requirement that $\mathrm{O}_{\text{Exh-DA}}$ must lead to a P(roperly) S(tronger) meaning + the assumption that, in checking for PS, we may also consider the presuppositions of the prejacent, if any. [4, 7]

Deriving variation w.r.t ignorance

⋆ Consider an exhaustification relative to just non-singleton subdomain alternatives, (5), and an exhaustification relative to just the singleton subdomain alternatives, (6). ($\Box_{\mathrm{S}}$ is a silent matrix-level epistemic necessity modal.)

(5) $\mathrm{O}_{\text{Exh-NonSg-DA}}\ \Box_{\mathrm{S}}(p \vee q \vee r) = \Box_{\mathrm{S}}(p \vee q \vee r) \wedge \underbrace{\neg \mathrm{O}\ \Box_{\mathrm{S}}(p \vee q)}_{=\Box_{\mathrm{S}}(p\vee q)\rightarrow\Box_{\mathrm{S}}(q\vee r)\vee\Box_{\mathrm{S}}(p\vee r)} \wedge \underbrace{\neg \mathrm{O}\ \Box_{\mathrm{S}}(q \vee r)}_{=\Box_{\mathrm{S}}(q\vee r)\rightarrow\Box_{\mathrm{S}}(p\vee q)\vee\Box_{\mathrm{S}}(p\vee r)} \wedge \underbrace{\neg \mathrm{O}\ \Box_{\mathrm{S}}(p \vee r)}_{=\Box_{\mathrm{S}}(p\vee r)\rightarrow\Box_{\mathrm{S}}(p\vee q)\vee\Box_{\mathrm{S}}(q\vee r)}$

(6) $\mathrm{O}_{\text{Exh-Sg-DA}}\ \Box_{\mathrm{S}}(p \vee q \vee r) = \Box_{\mathrm{S}}(p \vee q \vee r) \wedge \underbrace{\neg \mathrm{O}\ \Box_{\mathrm{S}}(p)}_{=\Box_{\mathrm{S}}p\rightarrow\Box_{\mathrm{S}}q\vee\Box_{\mathrm{S}}r} \wedge \underbrace{\neg \mathrm{O}\ \Box_{\mathrm{S}}(q)}_{=\Box_{\mathrm{S}}q\rightarrow\Box_{\mathrm{S}}p\vee\Box_{\mathrm{S}}r} \wedge \underbrace{\neg \mathrm{O}\ \Box_{\mathrm{S}}(r)}_{=\Box_{\mathrm{S}}r\rightarrow\Box_{\mathrm{S}}p\vee\Box_{\mathrm{S}}q}$

⋆ (5) can be verified by both a model of partial ignorance (e.g., $\Box p \wedge \neg\Box q \wedge \neg\Box r$) as well as a model of total ignorance ($\neg\Box p \wedge \neg\Box q \wedge \neg\Box r$); (6) can only be verified by a model of total ignorance ($\neg\Box p \wedge \neg\Box q \wedge \neg\Box r$).[a]
⋆ Assume all our items have to undergo exhaustification relative to their subdomain alternatives, but for *or* and SMNs this has to be done relative to the whole set (non-singletons + singletons), while for *some* NP_{SG} and CMNs this can be done relative to a pruned set consisting of just the non-singletons. Thus, the former can only be verified by a total ignorance model (due to the singletons) while the latter are compatible with a partial ignorance model.

[a] For domains of individuals as in the case of *or/some* NP_{SG}, both (5) and (6) can also be verified by models such as $\Box p \wedge \Box q \wedge \Box r$ or $\Box p \wedge \Box q \wedge \neg\Box r$, but we will assume those possibilities are ruled out by Scalar Implicatures (SIs) of the form $\neg\Box(p \wedge q \wedge r)$ or $\neg\Box(p \wedge q)$.

Deriving variation w.r.t anti-negativity

⋆ Consider exhaustification relative to subdomain alternatives for the case of an item in the scope of negation, (7), and for the case of an item in the antecedent of a conditional / restriction of a universal, (8). The latter environments also contain an existential presupposition; assume that too is part of the prejacent acted upon by O .

(7) $\mathrm{O}_{\text{Exh-DA}}\ \Box_{\mathrm{S}}\neg(p \vee q) = \Box_{\mathrm{S}}\neg(p \vee q)$ ($\mathrm{O}_{\text{Exh-DA}}$ vacuous)

(8) $\mathrm{O}_{\text{Exh-DA}}$ (assertive-and-presuppositional-component-of($\Box_{\mathrm{S}}\forall v[P(v) \vee Q(v) \rightarrow R(v)]$))
$= \mathrm{O}_{\text{Exh-DA}}\ (\underbrace{\Box_{\mathrm{S}}\forall v[P(v) \vee Q(v) \rightarrow R(v)]}_{\text{like (7)}} \wedge \underbrace{\Box_{\mathrm{S}}\exists v[P(v) \vee Q(v)]}_{\text{like (5)-(6)}})$ ($\mathrm{O}_{\text{Exh-DA}}$ not vacuous)

⋆ Assume, as before, that all our items have to undergo exhaustification relative to their subdomain alternatives, but for *or* and CMNs this can be vacuous, while for *some* NP_{SG} and SMNs it must obey Proper Strengthening. Thus, the former are fine under negation while the latter are not, and both are fine in the antecedent of a conditional / restriction of a universal.

Conclusion

- Similar ignorance and polarity sensitivity in disjunction, indefinites, and modified numerals.
- Uniform alternatives-and-exhaustification solution.
- Consequences for theories of modified numerals.

Truth conditions and alternatives: *or/some* NP_{SG}

(9) $\exists x \in \{a, b, \ldots\}[P(x)] \Leftrightarrow P(a) \vee P(b) \vee \ldots$
DA : Replace $D = \{a, b, \ldots\}$ with $D' \subset D$.
σA : Replace $\exists$ with $\forall$. (Can also do this for DA .)

Truth conditions and alternatives: CMNs/SMNs

(10) $[\![\mathrm{n}]\!] = n$ (type d)

(11) $[\![\mathrm{much}]\!]\ (n) = \lambda d\,.\, d \leq n$ (positive extent [8] of n)

(12) $[\![\mathrm{little}]\!]\ (n) = \lambda d\,.\, d \geq n$ (negative extent [8] of n)

(13) $[\![\text{more/less than n}]\!]$
$= [\![[\mathrm{comp}]]\!]\ ([\![\mathrm{much/little}]\!])(n)$
$= \lambda P_{\langle d,t\rangle}\,.\, \max(\lambda d\,.\, P(d)) \in [\![\mathrm{much/little}]\!]\ (n)$
E.g., $[\![\text{less than 3 P}]\!] = \max(\lambda d\,.\, P(d)) \in [\![\mathrm{little}]\!]\ (2)$.
DA : Replace $D = [\![\mathrm{much/little}]\!]\ (n)$ with $D' \subset D$.
σA : Replace n with m, where $m \in S$ (relevant scale).

(14) $[\![\text{at most/least n}]\!]$
$= [\![[\text{at-sup}]]\!]\ ([\![\mathrm{much/little}]\!])(n)$
$= \lambda P_{\langle d,t\rangle}\,.\, \max(\lambda d\,.\, P(d)) \in [\![\mathrm{much/little}]\!]\ (n)$
E.g., $[\![\text{at most 2 P}]\!] = \max(\lambda d\,.\, P(d)) \in [\![\mathrm{much}]\!]\ (2)$.
DA : Replace $D = [\![\mathrm{much/little}]\!]\ (n)$ with $D' \subset D$.
σA : Replace n with m, where $m \in S$ (relevant scale).

[1] Bart Geurts, Napoleon Katsos, Chris Cummins, Jonas Moons, and Leo Noordman. Scalar quantifiers: Logic, acquisition, and processing. *Language and cognitive processes*, 25(1):130–148, 2010.
[2] Anna Szabolcsi. Positive polarity–negative polarity. *NLLT*, 22(2):409–452, 2004.
[3] Teodora Mihoc and Kathryn Davidson. Testing a PPI analysis of superlative-modified numerals, 2017. Talk at XPrag 7.
[4] Gennaro Chierchia. *Logic in grammar: Polarity, free choice, and intervention.* Oxford University Press, Oxford, UK, 2013.
[5] Danny Fox. Free choice and the theory of scalar implicatures. In Uli Sauerland and P. Stateva, editors, *Presupposition and implicature in compositional semantics*, pages 71–120. Palgrave Macmillan, 2007.
[6] Moshe E Bar-Lev and Danny Fox. Universal free choice and innocent inclusion. In *Proceedings of SALT*, volume 27, pages 95–115, 2017.
[7] Andreea Nicolae. Deriving the positive polarity behavior of plain disjunction. *S&P*, 10, 2017.
[8] Chris Kennedy. *Projecting the adjective. The syntax and semantics of gradability and*