Ignorance and anti-negativity in the grammar: [*or*/*some*] and modified numerals

Puzzle. The disjunction *or* and the indefinite *some* NP_{SG} are similar: Given reference to the same domain of individuals, they are **truth-conditionally equivalent**, (1). Moreover, in a plain episodic context both give rise to speaker **ignorance** inferences, (2). However, they also differ in surprising ways w.r.t. **compatibility with certainty** – *some* NP_{SG} is compatible with positive or negative speaker certainty about a specific member of the domain whereas *or* is not, (3)-(4) – and **anti-negativity** – *or* can take scope below negation but *some* NP_{SG} can't, (5) (although both are fine in downward-entailing environments such as the antecedent of a conditional or the restriction of a universal). Strikingly, comparative-modified numerals (CMNs) and superlative-modified numerals (SMNs) exhibit the exact same type of similarity and variation, (1')-(5').

		compatibility with certainty	
		no	yes
anti-negativity	no	*or*	CMNs
	yes	SMNs	*some* NP_{SG}

(1) Jo called Penny or Quincey / some student. (= 1 iff $p \vee q$)

(2) Who did Jo call? Penny or Quincey. / Some student. ($\rightsquigarrow$ ignorance: $\Diamond p \wedge \Diamond q$)

(3) Jo called Penny. Therefore, he called #Penny, Quincey, or Ron / ✓some student.

(4) Jo called #Penny, Quincey, or Ron / ✓some student, but not Penny.

(5) Jo didn't call ✓Penny, Quincey, or Ron / #some student.

(1') Jo called less than 2 people / at most 1 person. (= 1 iff $0 \vee 1$)

(2') How many people did Jo call? Less than 2. / At most 1. ($\rightsquigarrow$ ignorance: $\Diamond 0 \wedge \Diamond 1$)

(3') Jo called 2 people. Therefore, he called ✓less than 3 / #at most 2.

(4') Jo called ✓less than 3 / #at most 2 people, but not 1.

(5') Jo didn't call ✓less than 3 / #at most 2 people.

Existing literature. Subsets of this puzzle have been noticed and/or analyzed in the literature (cf., e.g., [1], a.o., for anti-negativity in *some*; [2, 3], a.o., for ignorance in CMNs and SMNs; [4, 2, 5, 6], a.o., for anti-negativity in SMNs; [7, 8, 6], a.o., for the similarity between SMNs and disjunction). However, a theory that would capture all of (1)-(5) or all of (1')-(5'), and do so in a way that reflects the remarkable similarity between (1)-(5) and (1')-(5'), is still missing. The aim of this talk is to fill this gap.

A unified account of ignorance and anti-negativity in *or*/*some* NP_{SG} and CMNs/SMNs. Building on the alternatives-and-exhaustification approaches to epistemic indefinites and polarity sensitive items (cf. [9] and ref's therein), I account for the facts above as follows:

⋆ The truth conditions for (1)/(1') are (equivalent to) (6)/(6'). In particular, in a way that is commonly assumed for *or*/*some* NP_{SG} and will be defended for CMNs/SMNs, they make reference to a domain:

(6) $\exists x \in \mathbf{\{p,q\}}[C(j,x)]$
Abbreviated: $p \vee q$.

(6') $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in \mathbf{\{0,1\}}$
Abbreviated: $0 \vee 1$.

⋆ Replacing the domain in the truth conditions with its subsets yields subdomain alternatives, DA:

(7) $\{\exists x \in \mathbf{\{p\}}[C(j,x)], \exists x \in \mathbf{\{q\}}[C(j,x)]\}$
Abbreviated: $\{p, q\}$.

(7') $\{\max(\lambda d\,.\,\dots) \in \mathbf{\{0\}}, \max(\lambda d\,.\,\dots) \in \mathbf{\{1\}}\}$
Abbreviated: $\{0, 1\}$.

(There are also scalar alternatives, but for reasons of space I leave them out, as they are not crucial here.)

⋆ Alternatives are factored into meaning via a silent exhaustivity operator O . O asserts the prejacent and negates the non-entailed alternatives. The DA of *or*/*some* NP_{SG} /CMNs/SMNS must be factored in in a pre-exhaustified form, ExhDA (obtained by applying O to individual DA; I assume pre-exhaustification of a DA is done relative to other DA of the same size). Without an intervening operator, $\mathrm{O}_{\mathrm{ExhDA}}$ fails:

(8) $\mathrm{O}_{\mathrm{ExhDA}}\ (p \vee q)$
$= (p \vee q) \wedge \neg \underbrace{\underbrace{\mathrm{O}\ p}_{p \wedge \neg q}}_{p \to q} \wedge \neg \underbrace{\underbrace{\mathrm{O}\ q}_{q \wedge \neg p}}_{q \to p}$
a. $= (p \vee q) \wedge p \wedge q$ (clash w/ scalar implic)
b. $= (p \vee q) \wedge \neg p \wedge \neg q, = \bot$ (G-trivial)

(8') $\mathrm{O}_{\mathrm{ExhDA}}\ (0 \vee 1)$
$= (0 \vee 1) \wedge \neg \underbrace{\underbrace{\mathrm{O}\ 0}_{0 \wedge \neg 1}}_{0 \to 1} \wedge \neg \underbrace{\underbrace{\mathrm{O}\ 1}_{1 \wedge \neg 0}}_{1 \to 0}$
a. $= (0 \vee 1) \wedge 0 \wedge 1$ (logically impossible)
b. $= (0 \vee 1) \wedge \neg 0 \wedge \neg 1, = \bot$ (G-trivial)

⋆ An $\mathrm{O}_{\mathrm{ExhDA}}$ parse for episodic contexts is however possible if a null matrix level epistemic necessity modal $\Box_S$ (akin to the Gricean Bel_S 'the speaker believes …) is inserted as a last resort between $\mathrm{O}_{\mathrm{ExhDA}}$ and its prejacent. This yields ignorance, capturing (2)/(2').

(9) $\mathrm{O}_{\mathrm{ExhDA}}\ \Box_S\ (p \vee q)$

$$= \Box_S\ (p \vee q) \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ p}_{\Box_S\ p \to \Box_S\ q} \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ q}_{\Box_S\ q \to \Box_S\ p}$$

a. $= \Box_S\ (p \vee q) \wedge \Box_S\ p \wedge \Box_S\ q$ (clash w/ scalar implic)

b. $= \Box_S\ (p \vee q) \wedge \neg\Box_S\ p \wedge \neg\Box_S\ q$ $\boxed{\textbf{ignorance ✓}}$

(9') $\mathrm{O}_{\mathrm{ExhDA}}\ \Box_S\ (0 \vee 1)$

$$= \Box_S\ (0 \vee 1) \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ 0}_{\Box_S\ 0 \to \Box_S\ 1} \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ 1}_{\Box_S\ 1 \to \Box_S\ 0}$$

a. $= \Box_S\ (0 \vee 1) \wedge \Box_S\ 0 \wedge \Box_S\ 1$ (logically impossible)

b. $= \Box_S\ (0 \vee 1) \wedge \neg\Box_S\ 0 \wedge \neg\Box_S\ 1$ $\boxed{\textbf{ignorance ✓}}$

⋆ $\mathrm{O}_{\mathrm{ExhDA}}\ \Box_S$ above yielded ignorance about every element in the domain. How do we capture the contrasts relative to ignorance? *some NP_{SG}* /CMNs, but not *or*/SMNs, can prune their original DA set down to just a natural subset, e.g., just the non-singletons (NonSgDA) or just the singletons (SgDA). Exhaustification relative to the pruned sets yields compatibility with positive or negative certainty about a specific member of the domain, capturing (3)/(3')-(4)/(4'). I illustrate below for CMNs. (Assume this only happens for domains with more than 2 elements, and if forced by a non-total ignorance context.)

(10') $\mathrm{O}_{\mathrm{ExhNonSgDA}}\ \Box_S\ (0 \vee 1 \vee 2)$ $\boxed{\textbf{just NonSgDA} \Rightarrow \textbf{positive certainty about a specific element ✓}}$

$$= \underbrace{\Box_S\ (0 \vee 1 \vee 2) \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ (0 \vee 1)}_{\Box_S\ (0\vee 1) \to \Box_S\ (1 \vee 2) \vee \Box_S\ (0 \vee 2)} \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ (1 \vee 2)}_{\Box_S\ (1\vee 2) \to \Box_S\ (0 \vee 1) \vee \Box_S\ (0 \vee 2)} \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ (0 \vee 2)}_{\Box_S\ (0\vee 2) \to \Box_S\ (0 \vee 1) \vee \Box_S\ (1 \vee 2)}}_{\text{verified, e.g., by } \Box_S\ 2 (\wedge \neg\Box_S\ 0 / \Box_S\ \neg 0 \wedge \neg\Box_S\ 1 / \Box_S\ \neg 1)}$$

(11') $\mathrm{O}_{\mathrm{ExhSgDA}}\ \Box_S\ (0 \vee 1 \vee 2)$ $\boxed{\textbf{just SgDA} \Rightarrow \textbf{negative certainty about a specific element ✓}}$

$$= \underbrace{\Box_S\ (0 \vee 1 \vee 2) \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ (0)}_{\Box_S\ 0 \to \Box_S\ 1 \vee \Box_S\ 2} \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ (1)}_{\Box_S\ 1 \to \Box_S\ 0 \vee \Box_S\ 2} \wedge \underbrace{\neg\mathrm{O}\ \Box_S\ (2)}_{\Box_S\ 2 \to \Box_S\ 0 \vee \Box_S\ 1}}_{\text{verified, e.g., by } \Box_S\ \neg 1 (\wedge \neg\Box_S\ 0 \wedge \neg\Box_S\ 2)}$$

⋆ Turning to negation (w/ a 2-element domain, for simplicity), note that each ExhDA below is incompatible with the prejacent. It is therefore already excluded by it, and its negation adds nothing. *Or*/CMNs are fine with this result, but *some NP_{SG}* /SMNs require that exhaustification relative to their ExhDA lead to proper strengthening (PS). This explains their anti-negativity, capturing (5)/(5').

(12) $\mathrm{O}_{\mathrm{ExhDA}}\ \neg(p \vee q)$

$$= \neg(p \vee q) \wedge \neg\mathrm{O}\ \neg p \wedge \neg\mathrm{O}\ \neg q$$

$$= \neg(p \vee q) \wedge \neg \underbrace{(\neg p \wedge q)}_{\text{already excl.}} \wedge \neg \underbrace{(\neg q \wedge p)}_{\text{already excl.}}$$

$= \neg(p \vee q)$ $\boxed{\textbf{*no PS} \Rightarrow \textbf{anti-negativity ✓}}$

(12') $\mathrm{O}_{\mathrm{ExhDA}}\ \neg(0 \vee 1)$

$$= \neg(0 \vee 1) \wedge \neg\mathrm{O}\ \neg 0 \wedge \neg\mathrm{O}\ \neg 1$$

$$= \neg(0 \vee 1) \wedge \neg \underbrace{(\neg 0 \wedge 1)}_{\text{already excl.}} \wedge \neg \underbrace{(\neg 1 \wedge 0)}_{\text{already excl.}}$$

$= \neg(0 \vee 1)$ $\boxed{\textbf{*no PS} \Rightarrow \textbf{anti-negativity ✓}}$

Summary and outlook. I provide a unified alternatives-and-exhaustification account of ignorance and anti-negativity in *or*/*some NP_{SG}* and CMNs/SMNs that captures similarity and variation w.r.t. these phenomena both within and between these pairs. In the talk I will also discuss further connections/extensions to the existing literature on epistemic indefinites and polarity sensitive items, on the one hand, and numerals, on the other.

References. [1] Szabolcsi (2004) Positive polarity–negative polarity. [2] Geurts & Nouwen (2007) At least et al.: The semantics of scalar modifiers. [3] Cremers, Coppock, Dotlacil & Roelofsen (2017). Modified numerals: Two routes to ignorance. [4] Nilsen (2007) At least – Free choice and lowest utility. [5] Cohen & Krifka (2014) Superlative quantifiers and meta-speech acts. [6] Spector (2015) Why are class B modifiers global PPIs? [7] Büring (2008) The least at least can do. [8] Kennedy (2015) A "de-Fregean" semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. [9] Chierchia (2013) Logic in grammar: Polarity, free choice, and intervention.