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Presenter: Teodora Mihoc, Harvard University (tmihoc@fas.harvard.edu)
Talk: Ignorance and anti-negativity in the grammar: *or/some* $NP_{SG}$ and modified numerals
This document: Handout to accompany slides

# 1 *or/some* $NP_{SG}$

## 1.1 Truth conditions and alternatives

(1) Jo called $a$, $b$ or ...

a. $\exists x \in \{a, b, \dots\}[C(j, x)]$ (assertion)
b. $\{\exists x \in D'[C(j, x)] \mid D' \subset \{a, b, \dots\}\}$ (DA)
c. $\{\forall x \in \{a, b, \dots\}[C(j, x)]\}$ ($\sigma$A)
d. $\{\forall x \in D'[C(j, x)] \mid D' \subset \{a, b, \dots\}\}$ (D$\sigma$A)

(2) Jo called some student.

a. $\exists x \in [\![\text{student}]\!][C(j, x)]$ (assertion)
b. $\{\exists x \in D'[C(j, x)] \mid D' \subset [\![\text{student}]\!]\}$ (DA)
c. $\{\forall x \in [\![\text{student}]\!]\,[C(j, x)]\}$ ($\sigma$A)
d. $\{\forall x \in D'[C(j, x)] \mid D' \subset [\![\text{student}]\!]\}$ (D$\sigma$A)

**» examples**

(3) Jo called Alice or Bob / some student$_{\{\text{Alice, Bob}\}}$.

a. $\exists x \in \{a, b\}[C(j, x)]$ (assertion; abbr. $a \vee b$)
b. $\exists x \in \{a\}[C(j, x)]$ (singleton DA; abbr. $a$)
$\exists x \in \{b\}[C(j, x)]$ (singleton DA; abbr. $b$)
c. $\forall x \in \{a, b\}[C(j, x)]$ ($\sigma$A; abbr. $a \wedge b$)

(4) Jo called Alice, Bob, or Cindy / some student$_{\{\text{Alice, Bob, Cindy}\}}$.

a. $\exists x \in \{a, b, c\}[C(j, x)]$ (assertion; abbr. $a \vee b \vee c$)
b. $\exists x \in \{a\}[C(j, x)]$ (singleton DA; abbr. $a$)
$\exists x \in \{b\}[C(j, x)]$ (singleton DA; abbr. $b$)
$\exists x \in \{c\}[C(j, x)]$ (singleton DA; abbr. $c$)
$\exists x \in \{a, b\}[C(j, x)]$ (doubleton DA; abbr. $a \vee b$)
$\exists x \in \{a, c\}[C(j, x)]$ (doubleton DA; abbr. $a \vee c$)
$\exists x \in \{b, c\}[C(j, x)]$ (doubleton DA; abbr. $b \vee c$)
c. $\forall x \in \{a, b, c\}[C(j, x)]$ ($\sigma$A; abbr. $a \wedge b \wedge c$)
d. $\forall x \in \{a, b\}[C(j, x)]$ (doubleton D$\sigma$A; abbr. $a \wedge b$)
$\forall x \in \{a, c\}[C(j, x)]$ (doubleton D$\sigma$A; abbr. $a \wedge c$)
$\forall x \in \{b, c\}[C(j, x)]$ (doubleton D$\sigma$A; abbr. $b \wedge c$)

## 1.2 Exhaustification

⋆ Syntactically:

(5) $\text{O}_{\text{DA}}$(Jo called Alice or$_{[-\sigma,+\text{D}]}$ Bob / some$_{[-\sigma,+\text{D}]}$ student.)

(6) $\text{O}_{\sigma\text{A}}$(Jo called Alice or$_{[+\sigma,-\text{D}]}$ Bob / some$_{[+\sigma,-\text{D}]}$ student.)

⋆ Semantically:

(7) $[\![\text{O}_{\text{C}}(p)]\!]^{g,w} = [\![\text{p}]\!]^{g,w} \wedge \forall q \in [\![\text{p}]\!]^{\text{C}}\ [[\![\text{q}]\!]^{g,w} \rightarrow \lambda w'\,.\ [\![\text{p}]\!]^{g,w'} \subseteq q]$

E.g.,

(8) $\quad \mathrm{O}_{\mathrm{DA}}(a \vee b) = (a \vee b) \wedge \neg a \wedge \neg b, \; = \bot$ (G-trivial)

(9) $\quad \mathrm{O}_{\sigma\mathrm{A}}(a \vee b) = (a \vee b) \wedge \neg(a \wedge b)$ ($\rightsquigarrow$ *not and/every*)

$\star$ *or/some NP$_{SG}$* are exhaustified relative to pre-exhaustified DA, ExhDA:

(10) $\quad [\![\mathrm{p}]\!]^{\mathrm{ExhDA}} = \{\mathrm{O}(q) : q \in [\![\mathrm{p}]\!]^{\mathrm{DA}}\}$; e.g., $(a \vee b)^{\mathrm{ExhDA}} = \{\mathrm{O}a, \mathrm{O}b\}$

$\star$ Pre-exhaustification done relative to DA of the same size. E.g., for prejacent $(a \vee b \vee c)$, $\mathrm{O}a = a \wedge \neg b$; $\mathrm{O}(a \vee b) = (a \vee b) \wedge \neg(a \vee c) \wedge \neg(b \vee c)$.
$\star$ *or/some NP$_{SG}$* are by default exhaustified relative to both ExhDA and $\sigma$A; simplest version: $\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}$.

### 1.3 Capturing ignorance

(11) Jo called Alice or Bob / some student.
$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(a \vee b)$

a. $(a \vee b) \wedge$ (prejacent)

b. $\underbrace{\neg \underbrace{\mathrm{O}a}_{a \wedge \neg b}}_{a \to b} \wedge \underbrace{\neg \underbrace{\mathrm{O}b}_{b \wedge \neg a}}_{b \to a} \wedge$ (ExhDA-implicatures)

c. $\neg(a \wedge b)$ ($\sigma$A-implicature)

$= \bot$ (G-trivial)

(12) Jo may call Alice or Bob / some student.
$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(\Diamond(a \vee b))$

a. $\Diamond(a \vee b) \wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}(\Diamond a)}_{\Diamond a \wedge \neg \Diamond b}}_{\Diamond a \to \Diamond b} \wedge \underbrace{\neg \underbrace{\mathrm{O}(\Diamond b)}_{\Diamond b \wedge \neg \Diamond a}}_{\Diamond b \to \Diamond a} \wedge$

c. $\neg\Diamond(a \wedge b)$

$= \Diamond(a \vee b) \wedge \Diamond a \wedge \Diamond b \wedge \neg\Diamond(a \wedge b)$ (Free Choice)

(13) Jo must call Alice or Bob / some student.
$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(\Box(a \vee b))$

a. $\Box(a \vee b) \wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}(\Box a)}_{\Box a \wedge \neg \Box b}}_{\Box a \to \Box b} \wedge \underbrace{\neg \underbrace{\mathrm{O}(\Box b)}_{\Box b \wedge \neg \Box a}}_{\Box b \to \Box a} \wedge$

c. $\neg\Box(a \wedge b)$

$= \underbrace{\Box(a \vee b) \wedge \neg\Box a \wedge \neg\Box b}_{\Box(a \vee b) \wedge \Diamond a \wedge \Diamond b} \wedge \neg\Box(a \wedge b)$ (Free Choice)

(14) Jo called Alice or Bob / some student.
$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(\Box_{\mathrm{S}}(a \vee b))$

a. $\Box_{\mathrm{S}}(a \vee b) \wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}(\Box_{\mathrm{S}} a)}_{\Box_{\mathrm{S}} a \wedge \neg \Box_{\mathrm{S}} b}}_{\Box_{\mathrm{S}} a \to \Box_{\mathrm{S}} b} \wedge \underbrace{\neg \underbrace{\mathrm{O}(\Box_{\mathrm{S}} b)}_{\Box_{\mathrm{S}} b \wedge \neg \Box_{\mathrm{S}} a}}_{\Box_{\mathrm{S}} b \to \Box_{\mathrm{S}} a} \wedge$

c. $\neg\Box_{\mathrm{S}}(a \wedge b)$

$= \underbrace{\Box_{\mathrm{S}}(a \vee b) \wedge \neg\Box_{\mathrm{S}} a \wedge \neg\Box_{\mathrm{S}} b}_{\Box_{\mathrm{S}}(a \vee b) \wedge \Diamond_{\mathrm{S}} a \wedge \Diamond_{\mathrm{S}} b} \wedge \neg\Box_{\mathrm{S}}(a \wedge b)$ (epistemic Free Choice = ignorance)

(15) $\quad \mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}} \Box_{\mathrm{S}}(a \vee b \vee c)$

a. $\Box_S(a \vee b \vee c)\wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}\Box_S a}_{\Box_S a\wedge\neg\Box_S b\wedge\neg\Box_S c}}_{\Box_S a\rightarrow\Box_S b\vee\Box_S c} \wedge \underbrace{\neg \underbrace{\mathrm{O}\Box_S b}_{\Box_S b\wedge\neg\Box_S a\wedge\neg\Box_S c}}_{\Box_S b\rightarrow\Box_S a\vee\Box_S c} \wedge \underbrace{\neg \underbrace{\mathrm{O}\Box_S c}_{\Box_S c\wedge\neg\Box_S a\wedge\neg\Box_S b}}_{\Box_S c\rightarrow\Box_S a\vee\Box_S b} \wedge$

c. $\neg\Box_S(a \wedge b \wedge c)$

(M1) total ignorance / ‘no winner’:
$\neg\Box_S a \wedge \neg\Box_S b \wedge \neg\Box_S c$ ✓

(M2) partial ignorance with positive certainty / ‘one winner’:
$\Box_S a \wedge \neg\Box_S/\Box_S\neg b \wedge \neg\Box_S/\Box_S\neg c$ ✗
(Suppose $\Box_S a$. Then, if $\neg\Box_S b$ is true and $\neg\Box_S c$ is true, the second and the third implication can be true, but the first one cannot.)

(M3) partial ignorance with negative certainty / ‘one loser’:
$\Box_S\neg a \wedge \neg\Box_S b \wedge \neg\Box_S c$ ✓

(M4) no ignorance / ‘all winners’:
$\Box_S a \wedge \Box_S b \wedge \Box_S c$ ✗/✓
(Clash with the $\sigma$A-implicature. Possible if it is suspended.)

(16) $\mathrm{O}_{\text{ExhNonSgDA}+\sigma\text{A}}\Box_S(a \vee b \vee c)$

a. $\Box_S(a \vee b \vee c)\wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}\Box_S(a\vee b)}_{\Box_S(a\vee b)\wedge\neg\Box_S(a\vee c)\wedge\neg\Box_S(b\vee c)}}_{\Box_S(a\vee b)\rightarrow\Box_S(a\vee c)\vee\Box_S(b\vee c)} \wedge \underbrace{\neg \underbrace{\mathrm{O}\Box_S(a\vee c)}_{\Box_S(a\vee c)\wedge\neg\Box_S(a\vee b)\wedge\neg\Box_S(b\vee c)}}_{\Box_S(a\vee c)\rightarrow\Box_S(a\vee b)\vee\Box_S(b\vee c)} \wedge \underbrace{\neg \underbrace{\mathrm{O}\Box_S(b\vee c)}_{\Box_S(b\vee c)\wedge\neg\Box_S(a\vee b)\wedge\neg\Box_S(a\vee c)}}_{\Box_S(b\vee c)\rightarrow\Box_S(a\vee b)\vee\Box_S(a\vee c)} \wedge$

c. $\neg\Box_S(a \wedge b) \wedge \neg\Box_S(a \wedge c) \wedge \neg\Box_S(b \wedge c) \wedge \neg\Box_S(a \wedge b \wedge c)$

(M4) total ignorance / ‘no winner’:
$\neg\Box_S a \wedge \neg\Box_S b \wedge \neg\Box_S c$ ✓

(M2) partial ignorance with positive certainty / ‘one winner’:
$\Box_S a \wedge \neg\Box_S/\Box_S\neg b \wedge \neg\Box_S/\Box_S\neg c$ ✓

(M3) partial ignorance with negative certainty /‘one loser’:
$\Box_S\neg a \wedge \neg\Box_S b \wedge \neg\Box_S c$ ✗
(Consider, for example, the third implication. Suppose $\Box_S\neg a$ is true. Then, if $\neg\Box_S b \wedge \neg\Box_S c$ is also true, the whole consequent is false. This means that the implication can be true iff the antecedent $\Box_S(b\vee c)$ is also false. But this would contradict $\Box_S(a\vee b\vee c)\wedge\Box_S\neg a = \Box_S(b\vee c)$.)

(M4) no ignorance / ‘all winners’:
$\Box_S a \wedge \Box_S b \wedge \Box_S c$ ✗/✓
(Clash with the $\sigma$A-implicatures. Possible if they are suspended.)

$\star$ **Assumption:** To accommodate context, *some NP$_{SG}$*, but not *or*, can prune its DA-set to just SgDA or just NonSgDA. (To accommodate context, they can both also prune their $\sigma$A.)

## 1.4 Capturing polarity sensitivity

(17) Jo didn’t call Alice or Bob / some student$_{\{\text{Alice, Bob}\}}$.
$\mathrm{O}_{\text{ExhDA}+\sigma\text{A}}(\neg(a \vee b))$

a. $\neg(a \vee b)$

b. $\neg \underbrace{\mathrm{O}(\neg a)}_{\substack{\neg a\wedge\neg\neg b,\ =\neg a\wedge b \\ \text{already excluded by the prejacent}}} \wedge\neg \underbrace{\mathrm{O}(\neg b)}_{\substack{\neg b\wedge\neg\neg a,\ =\neg b\wedge a \\ \text{already excluded by the prejacent}}}$

c. $\neg \underbrace{(\neg(a \wedge b))}_{\text{already entailed by the prejacent}}$

⋆ **Assumption:** For presuppositional prejacents, O looks at the presupposition-enriched content (conjunction of assertive and presuppositional content) of the prejacent and of the alternatives. Then:

(18) If Jo called Alice or Bob / some student$_{\{\text{Alice, Bob}\}}$, she won.
Everyone who called Alice or Bob / some student$_{\{\text{Alice, Bob}\}}$ won.
$\text{O}^{\text{S}}_{\text{ExhDA+}\sigma\text{A}} \forall v[(a \vee b)_v \rightarrow W_v]$

a. $\forall v[(a \vee b)_v \rightarrow W_v] \wedge \exists v[(a \vee b)_v] \wedge$

b. $\underbrace{\neg \underbrace{\text{O}(\forall v[a_v \rightarrow W_v] \wedge \exists v[a_v])}_{(\forall v[a_v \rightarrow W_v] \wedge \exists v[a_v]) \wedge \neg(\forall v[b_v \rightarrow W_v] \wedge \exists v[b_v])}}_{(\forall v[a_v \rightarrow W_v] \wedge \exists v[a_v]) \rightarrow (\forall v[b_v \rightarrow W_v] \wedge \exists v[b_v])} \wedge \underbrace{\neg \underbrace{\text{O}(\forall v[b_v \rightarrow W_v] \wedge \exists v[b_v])}_{(\forall v[b_v \rightarrow W_v] \wedge \exists v[b_v]) \wedge \neg(\forall v[a_v \rightarrow W_v] \wedge \exists v[a_v])}}_{(\forall v[b_v \rightarrow W_v] \wedge \exists v[b_v]) \rightarrow (\forall v[a_v \rightarrow W_v] \wedge \exists v[a_v])}$

c. $\neg(\forall v[(a \wedge b) \rightarrow W_v] \wedge \exists v[(a \wedge b)_v \rightarrow W_v]$

(M1) (a) $\wedge \exists v[a_v] \wedge \exists v[b_v]$ (cf. $\text{O}_{\text{ExhDA+}\sigma\text{A}}(\Diamond(a \vee b))$, Free Choice)

(M2) (a) $\wedge \neg\Box\exists v[a_v] \wedge \neg\Box\exists v[b_v]$ (cf. $\text{O}_{\text{ExhDA+}\sigma\text{A}}(\Box_{\text{S}}(a \vee b))$, Free Choice)

⋆ **Assumption:** *some NP$_{SG}$*, but not *or*, requires that $\text{O}_{\text{ExhDA}}$ must lead to a properly stronger meaning.

# 2 BNs/CMNs/SMNs

## 2.1 Truth conditions and alternatives (for CMNs/SMNs, see tree on p. 8)

(19) $n$ people quit.

a. $\exists x[|x| = n \wedge P(x) \wedge Q(x)]$ (assertion)
b. $-$ (no DA)
c. $\{\exists x[|x| = m \wedge P(x) \wedge Q(x)] \mid m \in S\}$ ($\sigma$A)
d. $-$ (no D$\sigma$A b/c no DA)

(20) $[\![\text{much}]\!] = \lambda n \,.\, \lambda d \,.\, d \leq n$

e.g., $[\![\text{much}]\!]\,(3) = \lambda d \,.\, d \leq 3$

(21) $[\![\text{little}]\!] = \lambda n \,.\, \lambda d \,.\, d \geq n$

e.g., $[\![\text{little}]\!]\,(3) = \lambda d \,.\, d \geq 3$

(22) More/less than $n$ people quit.

a. $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(n)}$ (assertion)
b. $\{\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset \overline{[\![\text{much/little}]\!]\,(n)}\}$ (DA)
c. $\{\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overline{[\![\text{much/little}]\!]\,(m)} \mid m \in S\}$ ($\sigma$A)
d. $-$ (no D$\sigma$A b/c impossible or identical to existing $\sigma$A)

(23) At most/least $n$ people quit.

a. $\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(n)$ (assertion)
b. $\{\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(n)\}$ (DA)
c. $\{\max(\lambda d \,.\, \exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ ($\sigma$A)
d. $-$ (no D$\sigma$A b/c impossible or identical to existing $\sigma$A)

**» examples**

(24) Three people quit.

a. $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (assertion; abbr. $3 \vee 4 \vee \ldots$)
b. $-$ (no DA)

c. $\dots$ $\dots$
$\exists x[|x| = 2 \wedge P(x) \wedge Q(x)]$ ($\sigma$A; abbr. $2 \vee 3 \vee \dots$)
$\exists x[|x| = 4 \wedge P(x) \wedge Q(x)]$ ($\sigma$A; abbr. $4 \vee 5 \vee \dots$)
$\dots$ $\dots$

(25) Less than two people quit. / At most one person quit.

a. $\max \in \underbrace{\overline{[\![\text{little}]\!]\,(2)}/\,[\![\text{much}]\!]\,(1)}_{\{0,1\}}$ (assertion; abbr. $0 \vee 1$)
b. $\max \in \{0\}$ (singleton DA; abbr. 0)
$\max \in \{1\}$ (singleton DA; abbr. 1)
c. $\max \in \underbrace{\overline{[\![\text{little}]\!]\,(1)}/\,[\![\text{much}]\!]\,(0)}_{\{0\}}$ ($\sigma$A; abbr. 0)
$\max \in \underbrace{\overline{[\![\text{little}]\!]\,(3)}/\,[\![\text{much}]\!]\,(2)}_{\{0,1,2\}}$ ($\sigma$A; abbr. $0 \vee 1 \vee 2$)

(26) Less than three people quit. / At most two people quit.

a. $\max \in \underbrace{\overline{[\![\text{little}]\!]\,(3)}/\,[\![\text{much}]\!]\,(2)}_{\{0,1,2\}}$ (assertion; abbr. $0 \vee 1 \vee 2$)
b. $\max \in \{0\}$ (singleton DA; abbr. 0)
$\max \in \{1\}$ (singleton DA; abbr. 1)
$\max \in \{2\}$ (singleton DA; abbr. 2)
$\max \in \{0, 1\}$ (doubleton DA; abbr. $0 \vee 1$)
$\max \in \{0, 2\}$ (doubleton DA; abbr. $0 \vee 2$)
$\max \in \{1, 2\}$ (doubleton DA; abbr. $1 \vee 2$)
c. $\max \in \underbrace{\overline{[\![\text{little}]\!]\,(1)}/\,[\![\text{much}]\!]\,(0)}_{\{0\}}$ ($\sigma$A; abbr. 0)
$\max \in \underbrace{\overline{[\![\text{little}]\!]\,(2)}/\,[\![\text{much}]\!]\,(1)}_{\{0,1\}}$ ($\sigma$A; abbr. $0 \vee 1$)
$\max \in \underbrace{\overline{[\![\text{little}]\!]\,(4)}/\,[\![\text{much}]\!]\,(3)}_{\{0,1,2,3\}}$ ($\sigma$A; abbr. $0 \vee 1 \vee 2 \vee 3$)
$\dots$ $\dots$

## 2.2 Exhaustification

Same as for *or*/*some* $NP_{SG}$.

## 2.3 Scalar implicatures – reasons to rehabilitate them

⋆ Conceptual generality: *or*/*some* $NP_{SG}$/CMNs/SMNs all entail one bound, $\sigma$A-implicate another.
⋆ Empirical predictions generally good, and in some cases unique to this approach (indirect $\sigma$A-implicatures).
⋆ Once we dig deeper, the problematic cases in fact never even arise (see (33) and (36) below).

## 2.4 Capturing ignorance

(27) Jo called less than two people / at most one person.
$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(0 \vee 1)$

a. $(0 \vee 1)\wedge$ (prejacent)
b. $\wedge \neg \underbrace{\underbrace{\mathrm{O}0}_{0\wedge\neg 1}}_{0\to 1} \wedge \neg \underbrace{\underbrace{\mathrm{O}1}_{1\wedge\neg 0}}_{1\to 0} \wedge$ (ExhDA-implicatures)

c. $\neg 0$ ($\sigma$A-implicatures)

$= \bot$ (G-trivial)

(28) Jo may call less than two people / at most one person.

$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(\Diamond(0 \vee 1))$

a. $\Diamond(0 \vee 1)\wedge$

b. $\neg \underbrace{\underbrace{\mathrm{O}(\Diamond 0)}_{\Diamond 0\wedge\neg\Diamond 1}}_{\Diamond 0\to\Diamond 1} \wedge \neg \underbrace{\underbrace{\mathrm{O}(\Diamond 1)}_{\Diamond 1\wedge\neg\Diamond 0}}_{\Diamond 1\to\Diamond 0} \wedge$

c. $\neg\Diamond 0$

$= \Diamond(0 \vee 1) \wedge \Diamond 0 \wedge \Diamond 1 \cancel{\wedge\neg\Diamond 0}$ (after default $\sigma$A-pruning (see end of section), Free Choice)

(Other exhaustification parses, e.g., $\mathrm{O}_{\sigma\mathrm{A}}\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(a \vee b)$, can also yield stronger results.)

(29) Jo must call less than two people / at most one person.

$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(\Box(0 \vee 1))$

a. $\Box(0 \vee 1)\wedge$

b. $\neg \underbrace{\underbrace{\mathrm{O}(\Box 0)}_{\Box 0\wedge\neg\Box 1}}_{\Box 0\to\Box 1} \wedge \neg \underbrace{\underbrace{\mathrm{O}(\Box 1)}_{\Box 1\wedge\neg\Box 0}}_{\Box 1\to\Box 0} \wedge$

c. $\neg\Box 0$

$= \underbrace{\Box(0 \vee 1) \wedge \neg\Box 0 \wedge \neg\Box 1}_{\Box(0\vee 1)\wedge\Diamond 0\wedge\Diamond 1} \wedge\neg\Box 0$ (Free Choice)

(30) Jo called less than two people / at most one person.

$\mathrm{O}_{\mathrm{ExhDA}+\sigma\mathrm{A}}(\Box_S(0 \vee 1))$

a. $\Box_S(0 \vee 1)\wedge$

b. $\neg \underbrace{\underbrace{\mathrm{O}(\Box_S 0)}_{\Box_S 0\wedge\neg\Box_S 1}}_{\Box_S 0\to\Box_S 1} \wedge \neg \underbrace{\underbrace{\mathrm{O}(\Box_S 1)}_{\Box_S 1\wedge\neg\Box_S 0}}_{\Box_S 1\to\Box_S 0} \wedge$

c. $\neg\Box_S 0$

$= \underbrace{\Box_S(0 \vee 1) \wedge \neg\Box_S 0 \wedge \neg\Box_S 1}_{\Box_S(0\vee 1)\wedge\Diamond 0\wedge\Diamond 1} \wedge\neg\Box_S 0$ (epistemic Free Choice = ignorance)

(31) $\mathrm{O}_{\mathrm{ExhSgDA}+\sigma\mathrm{A}}\Box_S(0 \vee 1 \vee 2)$

a. $\Box_S(0 \vee 1 \vee 2)\wedge$

b. $\neg \underbrace{\underbrace{\mathrm{O}\Box_S 0}_{\Box_S 0\wedge\neg\Box_S 1\wedge\neg\Box_S 2}}_{\Box_S 0\to\Box_S 1\vee\Box_S 2} \wedge \neg \underbrace{\underbrace{\mathrm{O}\Box_S 1}_{\Box_S 1\wedge\neg\Box_S 0\wedge\neg\Box_S 1}}_{\Box_S 1\to\Box_S 0\vee\Box_S 2} \wedge \neg \underbrace{\underbrace{\mathrm{O}\Box_S 2}_{\Box_S 2\wedge\neg\Box_S 0\wedge\neg\Box_S 1}}_{\Box_S 2\to\Box_S 0\vee\Box_S 1} \wedge$

c. $\neg\Box_S 0 \wedge \neg\Box_S(0 \vee 1)$

(M1) total ignorance / ‘no winner’:

$\neg\Box_S 0 \wedge \neg\Box_S 1 \wedge \neg\Box_S 2$ ✓

(M2) partial ignorance with positive certainty / ‘one winner’:

$\Box_S 0 \wedge \cancel{\neg\Box_S /}\Box_S\neg 1 \wedge \cancel{\neg\Box_S /}\Box_S\neg 2$ ✗

(The first implication would end up false.)

(M3) partial ignorance with negative certainty / ‘one loser’:

$\Box_S\neg 0 \wedge \neg\Box_S 1 \wedge \neg\Box_S 2$ ✗/✓

(For $\Box_S\neg 2$, in conjunction with the prejacent, clash with the $\sigma$A-implicature $\neg\Box_S(0 \vee 1)$, possible if it is suspended.)

(M4) no ignorance / ‘all winners’:

$\Box_S 0 \wedge \Box_S 1 \wedge \Box_S 2$ ✗
(Impossible because of the nature of the domain.)

(32)

$O_{\text{ExhNonSgDA}+\sigma\text{A}}\Box_S(0 \vee 1 \vee 2)$

a. $\Box_S(0 \vee 1 \vee 2)\wedge$

b. $\underbrace{\neg \underbrace{O\Box_S(0\vee 1)}_{\Box_S(0\vee1)\wedge\neg\Box_S(0\vee2)\wedge\neg\Box_S(1\vee2)}}_{\Box_S(0\vee1)\rightarrow\Box_S(0\vee2)\vee\Box_S(1\vee2)} \wedge \underbrace{\neg \underbrace{O\Box_S(0\vee 2)}_{\Box_S(0\vee2)\wedge\neg\Box_S(0\vee1)\wedge\neg\Box_S(1\vee2)}}_{\Box_S(0\vee2)\rightarrow\Box_S(0\vee1)\vee\Box_S(1\vee2)} \wedge \underbrace{\neg \underbrace{O\Box_S(1\vee 2)}_{\Box_S(1\vee2)\wedge\neg\Box_S(0\vee1)\wedge\neg\Box_S(0\vee2)}}_{\Box_S(1\vee2)\rightarrow\Box_S(0\vee1)\vee\Box_S(0\vee2)} \wedge$

c. $\neg\Box_S 0 \wedge \neg\Box_S(0 \vee 1)$

(M1) total ignorance / 'no winner':
$\neg\Box_S 0 \wedge \neg\Box_S 1 \wedge \neg\Box_S 2$ ✓

(M2) partial ignorance with positive certainty / 'one winner':
$\Box_S 0 \wedge \cancel{\neg\Box_S}\Box_S\neg 1 \wedge \cancel{\neg\Box_S}\Box_S\neg 2$ ✗/✓
(Clash with the $\sigma$A-implicature $\neg\Box_S 0$, possible if it suspended.)

(M3) partial ignorance with negative certainty / 'one loser':
$\Box_S\neg 0 \wedge \neg\Box_S 1 \wedge \neg\Box_S 2$ ✗
(Consider the third implication. Suppose $\Box_S\neg 0$ is true. If $\neg\Box_S 1 \wedge \neg\Box_S 2$ is true also, then the whole consequent is false, so for the implication to be true, the antecedent $\Box_S(1 \vee 2)$ must be false. But this would contradict the conjunction of the prejacent $\Box_S(0 \vee 1 \vee 2)$ with our assumption $\Box_S\neg 0$, which entails $\Box_S(1 \vee 2)$.)

(M4) no ignorance / 'all winners':
$\Box_S 0 \wedge \Box_S 1 \wedge \Box_S 2$ ✗
(Impossible because of the nature of the domain.)

⋆ **Assumption:** To accommodate context, CMNs, but not SMNs, can prune their DA-set to just SgDA or just NonSgDA. (To accommodate context or to avoid clash with ExhDA, they can both also prune their $\sigma$A.)

**'Exactly' scalar implicature is never in fact generated**

(33) Jo called less than three people / at most two people. $\not\rightsquigarrow$ 'exactly 2'
$O_{\text{ExhDA}}(\Box_S O_{\sigma\text{A}}(0 \vee 1 \vee 2))$

a. $\Box_S O_{\sigma\text{A}}(0 \vee 1 \vee 2)\wedge$

b. $\neg O\Box_S 0 \wedge \neg O\Box_S 1 \wedge \neg O\Box_S 2 \wedge \neg O\Box_S(0 \vee 1) \wedge \neg O\Box_S(1 \vee 2) \wedge \neg O\Box_S(0 \vee 2)$

$= \underbrace{\underbrace{\underbrace{(a)}_{\Box_S((0\vee1\vee2)\wedge\neg(0\vee1))}}_{=\Box_S 2} \wedge \underbrace{(b)}_{\neg\Box_S 0\wedge\neg\Box_S 1\wedge\neg\Box_S 2}}_{\perp}$ ($\perp$ resolved by default $\sigma$A-pruning)

## 2.5 Capturing polarity sensitivity

(34) Jo didn't call less than two / at most one people.
$O_{\text{ExhDA}+\sigma\text{A}}(\neg(0 \vee 1))$

a. $\neg(0 \vee 1)$

b. $\neg \underbrace{O(\neg 0)}_{\substack{\neg0\wedge\neg\neg1,\ =\neg0\wedge1 \\ \text{already excluded by the prejacent}}} \wedge\neg \underbrace{O(\neg 1)}_{\substack{\neg1\wedge\neg\neg0,\ =\neg1\wedge0 \\ \text{already excluded by the prejacent}}}$

c. $\underbrace{\neg \underbrace{(\neg(0 \vee 1 \vee 2))}_{\text{not entailed by the prejacent}}}_{0\vee 1\vee 2}$

⋆ **Assumption:** O looks at presupposition-enriched prejacent and alternatives. Then:

(35) If Jo called less than two / at most one people he won.
Everyone who called less than two / at most one people won.
$\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}+\sigma\mathrm{A}} \forall v[(0 \vee 1)_v \to W_v]$

a. $\forall v[(0 \vee 1)_v \to W_v] \wedge \exists v[(0 \vee 1)_v] \wedge$

b. $\underbrace{\neg \underbrace{\mathrm{O}(\forall v[0_v \to W_v] \wedge \exists v[0_v])}_{(\forall v[0_v \to W_v] \wedge \exists v[0_v]) \wedge \neg(\forall v[1_v \to W_v] \wedge \exists v[1_v])}}_{(\forall v[0_v \to W_v] \wedge \exists v[0_v]) \to (\forall v[1_v \to W_v] \wedge \exists v[1_v])} \wedge \underbrace{\neg \underbrace{\mathrm{O}(\forall v[1_v \to W_v] \wedge \exists v[1_v])}_{(\forall v[1_v \to W_v] \wedge \exists v[1_v]) \wedge \neg(\forall v[0_v \to W_v] \wedge \exists v[0_v])}}_{(\forall v[1_v \to W_v] \wedge \exists v[1_v]) \to (\forall v[0_v \to W_v] \wedge \exists v[0_v])}$

c. $\neg(\forall v[(0 \vee 1 \vee 2)_v \to W_v] \wedge \exists v[(0 \vee 1 \vee 2)_v \to W_v]$

⋆ **Assumption:** SMNs, but not CMNs, require that $\mathrm{O}_{\mathrm{ExhDA}}$ must lead to a properly stronger meaning.

**'Exactly' scalar implicature is never in fact generated**
⋆ The $\sigma$A of, e.g., 3 under negation are not just $\{\ldots, \neg 2, \neg 4, \ldots\}$ but also $\{\ldots, 2, 4, \ldots\}$. Negating all the non-entailed $\sigma$A leads to $\perp$. With last resort insertion of $\Box_{\mathrm{S}}$, it leads to ignorance.

(36) Jo didn't call three / more than two / # at least three people. $\not\rightsquigarrow$ 'exactly 2'
$\mathrm{O}_{\sigma\mathrm{A}} \Box_{\mathrm{S}} \neg(3 \vee 4 \vee \ldots)$

a. $\Box_{\mathrm{S}} \neg(3 \vee 4 \vee \ldots) \wedge$

b. $\neg \Box_{\mathrm{S}} \neg(2 \vee \ldots) \wedge \neg \Box_{\mathrm{S}} \neg(1 \vee \ldots) \wedge \ldots$ (traditional $\sigma$A)

c. $\neg \Box_{\mathrm{S}} (2 \vee \ldots) \wedge \neg \Box_{\mathrm{S}} (1 \vee \ldots) \wedge \ldots$ (new $\sigma$A, obtained by deleting $\neg$)

'In all the worlds compatible with what the speaker believes the relevant number is not three or more but the speaker is not sure which one of the remaining numbers (0 or 1 or 2) it is.'

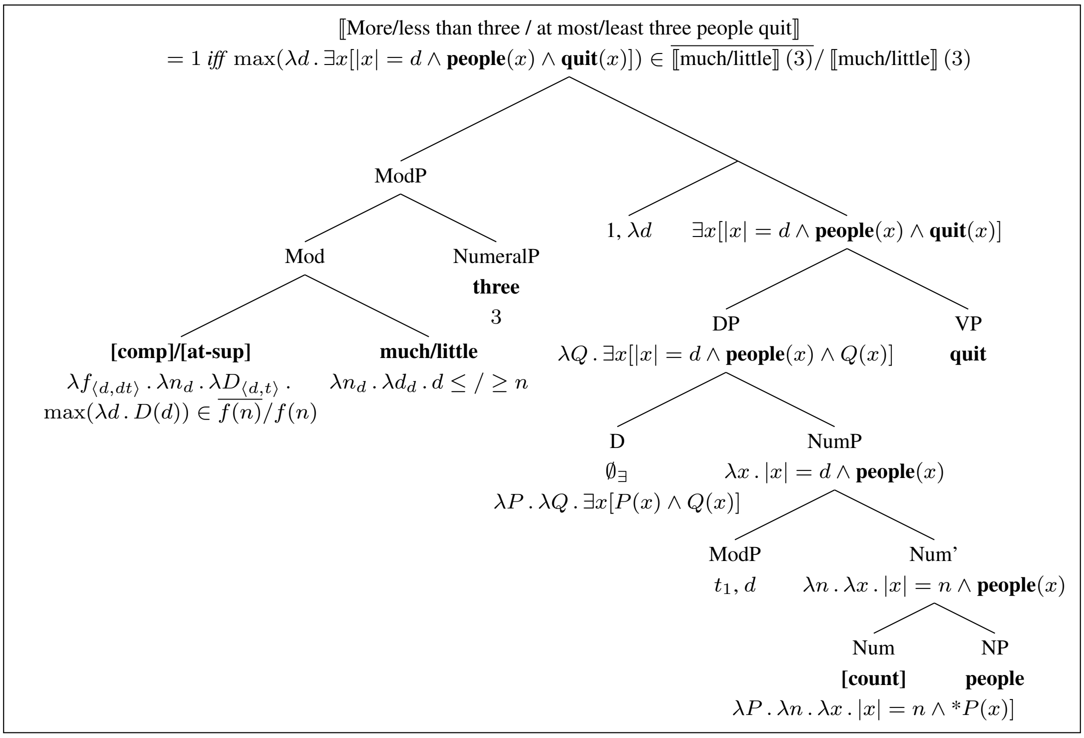


Figure 1: The syntax and semantics of CMNs and SMNs.