Modified numerals and polarity sensitivity:

Between O(nly)$_{DA}$ and E(ven)$_{SA}$

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Disclaimer

- By “modified numerals” I have in mind both
  - comparative-modified numerals (CMNs) and
  - superlative-modified numerals (SMNs).

- However, for reasons of time, I will focus just on SMNs.

  (The analysis for CMNs is parallel, except in certain key points, which will be noted.)

Outline

Introduction

The mostly understood patterns

- Data
- Existing literature
- Analysis

The mostly mysterious patterns

- Data
- Existing literature
- Analysis

Conclusion and outlook

SMNs exhibit many interesting patterns

- ignorance / modal variation / quantificational variability: free choice (FC)

  Jo solved at least 3 problems. ⇝ speaker ignorance

- anti-negativity / positive polarity: polarity sensitivity 1 (POL 1)

  Jo didn't solve # at least 3 problems.
  If Jo solved at least 3 problems, she passed.

- scalar implicatures (SI)

  Jo solved at least 3 problems. ⇝ Jo didn't solve at least, e.g., 5.
  If Jo solved at least 3 problems, she passed. ⇝ ¬ If Jo solved, e.g., at least 2…

- sensitivity to polarity in other ways: polarity sensitivity 2 (POL2)

  If Jo solved at most 3 problems, ? she passed.
  If Jo solved at least 3 problems, ? she failed.
  If Jo didn't solve at least 3 problems, ? she passed.

Some mostly *understood*, others mostly *mysterious*

- ignorance / modal variation / quantificational variability: free choice (FC) ✓

  Jo solved at least 3 problems. $\rightsquigarrow$ speaker ignorance

- anti-negativity / positive polarity: polarity sensitivity 1 (POL 1) ✓

  Jo didn't solve # at least 3 problems.
  If Jo solved at least 3 problems, she passed.

- scalar implicatures (SI) ✓

  Jo solved at least 3 problems. $\rightsquigarrow$ $\neg$ Jo solved, e.g., at least 5 problems.
  If Jo solved at least 3 problems, she passed. $\rightsquigarrow$ $\neg$ If Jo solved, e.g., at least 2...

- sensitivity to polarity in other ways: polarity sensitivity 2 (POL2) ?

  If Jo solved at most 3 problems, ? she passed.
  If Jo solved at least 3 problems, ? she failed.
  If Jo didn't solve at least 3 problems, ? she passed.

In this talk we will discuss ...

- the mostly understood patterns—FC, POL1, SI based on my previous work
- the mostly mysterious patterns—POL2 new to this talk
- interactions of POL2 with FC, POL1, and SI new to this talk

Preview of proposal

SMNs naturally activate both subdomain alternatives (DA) and scalar alternatives (SA).

These are factored into meaning via the silent exhaustivity operators O(nly) & E(ven).

Exhaustification via O

- relative to the pre-exhaustified non-entailed DA → FC and POL1
- relative to the non-entailed SA → SI

Exhaustification via E

- relative to the pre-exhaustified *entailed* SA → POL2 the main novelty today!

We find effects of all even in a sentence as simple as *Jo solved at least 3 problems*.

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(1) Jo solved **at least 3** problems. $\Diamond_S 3 \wedge \Diamond_S 4 \wedge \ldots$

(2) Jo solved # **at least 3** problems; to be more precise, **5**. # spec. pos. knowledge

(3) Jo solved # **at least 3** problems, but **not 5**. # spec. neg. knowledge

(4) Jo must solve **at least 3** problems. $\Diamond 3 \wedge \Diamond 4 \wedge \ldots$

(5) Everyone solved **at least 3** problems. $\exists x_3 \wedge \exists x_4 \wedge \ldots$ / $\Diamond_S 3 \wedge \Diamond_S 4 \wedge \ldots$

(6) Jo may solve **at least 3** problems. $\Diamond 3 \wedge \Diamond 4 \wedge \ldots$ / $\Diamond_S 3 \wedge \Diamond_S 4 \wedge \ldots$

(7) Someone solved **at least 3** problems. $\Diamond_S 3 \wedge \Diamond_S 4 \wedge \ldots$

(8) Jo did**n't** solve # **at least 3** problems. — # not > SMN

(9) **Nobody** solved # **at least 3** problems. — # nobody > SMN

(10) Jo passed **without** solving # **at least 3** problems. — # without > SMN

(11) **Few** students solved # **at least 3** problems. — # few > SMN

(12) Jo **rarely** solved # **at least 3** problems. — # rarely > SMN

(13) **If** Jo solved **✓at least 3** problems, she **passed.** — ✓if > SMN

(14) **Every**one who solved **✓at least 3** problems **passed**. — ✓every > SMN

(15) **Only** kids aged **✓at least 3** can attend kindergarten. — ✓only > SMN

(16) Jo solved **at least 3** problems. # $\Box_S \neg$ at least 4 $\rightarrow$ exactly 3

(17) Jo didn't solve # **at least 3** problems. # $\Box_S$ at least 2 $\rightarrow$ exactly 2

(18) Jo solved **at least 3** problems. ✓$\Box_S \neg$, e.g., at least 5

(19) Jo didn't solve # **at least 3** problems. ✓$\Box_S$, e.g., at least 1

(20) Jo must solve **at least 3** problems. ✓$\neg\Box$ at least 4

(21) Everyone solved **at least 3** problems. ✓$\neg\forall x$[at least 4_x]

(22) If Jo solved **at least 3** problems, she passed. ✓$\neg\forall w$[at least 2_w $\rightarrow$]

Mostly known:

- Basic patterns.

Mostly not recognized:

- These are all FC effects.[1]
- The effects are parallel in SMNs and CMNs, only in CMNs weaker.[2]
- SMNs are total FC items whereas CMNs are a type of partial FC items.[3]

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[1] For ignorance as null epistemic FC, cf. [Kratzer and Shimoyama, 2002], [Chierchia, 2013] for epistemic indefinites.

[2] For exp. evidence of in/compatibility with specific knowledge in SMNs/CMNs, cf. [Geurts and Nouwen, 2007, Geurts et al., 2010], [Cummins and Katsos, 2010, Cremers and Chemla, 2017] and for evidence of FC in CMNs cf. [Westera and Brasoveanu, 2014], [Cremers and Chemla, 2017] for ignorance and [Alexandropoulou et al., 2015] for quantificational variability.

[3] For first comparison of ignorance in SMNs to FC effects in epistemic indefinites, cf. [Nouwen, 2015]. For arguments for total vs. partial, cf. [Mihoc, 2019, Mihoc, 2020].

Mostly known:

- Basic contrasts. Also the fact that they are not present in CMNs.[4]

Mostly not recognized:

- Contrasts as in PPIs.[5]
- Contrasts sensitive to the non-truth-conditional content of the DE environment.[6]

[4] [Geurts and Nouwen, 2007], [Nilsen, 2007, Cohen and Krifka, 2014]. For exp. evidence, cf. also [Mihoc and Davidson, 2017].

[5] For explicit comparison of SMNs to PPIs, cf. [Spector, 2014, Spector, 2015, Mihoc, 2019, Mihoc, 2020].

[6] For explicit observations that PPIs exhibit the same sensitivity to non-truth-conditional content as strong NPIs, only in the opposite direction, cf. [Spector, 2014], [Nicolae, 2017].

Mostly known:

- Basic problem in plain contexts and non-problem under *must* or *every*.

Mostly not recognized:

- Problem and non-problem patterns both include both direct and indirect SI.[7]
- Solutions that abandon classic [Horn, 1972] alternatives can't capture indirect SI.[8]
- Classic SI predictions ok everywhere except where they lead to exact meaning.[9]

[7] For indirect SI in general, cf. [Chierchia, 2004] for bare numerals and other items. For same problem / non-problem with indirect SI, cf. [Spector, 2013] for bare numerals.

[8] [Mihoc, 2019], [Mihoc, 2020].

[9] Cf. [Mayr, 2013]

Analysis: Basics[10]

(to appendix »)

- SMNs contain reference to both a scalar element and a domain based on it.

(23) At most/least n people quit.

$$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(n)}^{\{\dots,n-1,n\}/\{n,n+1,\dots\}}$$

- Replacing the scalar element with its scalemates yields scalar alternatives (SA).

(24) $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$

- Replacing the domain with its subsets yields subdomain alternatives (DA).

(25) $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(n)\}$

- Alternatives used via the contradiction-based silent exhaustivity operator O(nly).

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[10] Cf. [Mihoc, 2019, Mihoc, 2020], using insights from [Kennedy, 1997]'s extent analysis of gradable adjectives and from the existing alternatives-and-exhaustification solutions to numerals [Büring, 2008, Kennedy, 2015], [Spector, 2015], [Schwarz, 2016], [Nouwen, 2015], disjunction [Fox, 2007, Nicolae, 2017], and indefinites [Alonso-Ovalle and Menéndez-Benito, 2010], [Chierchia, 2013].

(26) Jo solved at most 2 problems.

$$0 \quad 1 \quad 2 \qquad \text{(DA)}$$

$$0 \lor 1 \quad 0 \lor 2 \quad 1 \lor 2$$

$$\downarrow$$

$$0 \rightarrow 0 \lor 1 \rightarrow \mathbf{0} \lor \mathbf{1} \lor \mathbf{2} \rightarrow \cdots \lor 3 \rightarrow \ldots \text{(SA)}$$

assertion in boldface
DA in red
SA in blue
arrows indicate direction of entailment

(27) Jo solved at least 3 problems.

$$3 \quad 4 \quad 5 \quad \ldots \qquad \text{(DA)}$$

$$3 \lor 4 \quad 3 \lor 4 \lor 5 \quad 4 \lor 7 \lor 8 \lor 10 \quad \ldots$$

$$\downarrow$$

$$\ldots \leftarrow 2 \lor \ldots \leftarrow \mathbf{3} \lor \mathbf{4} \lor \ldots \leftarrow 4 \lor \ldots \leftarrow \ldots \text{(SA)}$$

(28) Jo solved at most 2 problems.

$$0 \quad 1 \quad 2 \qquad \text{(DA)}$$
$$0 \vee 1 \quad 0 \vee 2 \quad 1 \vee 2$$
$$\downarrow$$
$$0 \rightarrow 0 \vee 1 \rightarrow \mathbf{0} \vee \mathbf{1} \vee \mathbf{2} \rightarrow \cdots \vee 3 \rightarrow \ldots \text{(SA)}$$

(29) Jo solved at least 3 problems.

$$3 \quad 4 \quad 5 \quad \ldots \qquad \text{(DA)}$$
$$3 \vee 4 \quad 3 \vee 4 \vee 5 \quad 4 \vee 7 \vee 8 \vee 10 \quad \ldots$$
$$\downarrow$$
$$\ldots \leftarrow 2 \vee \ldots \leftarrow \mathbf{3} \vee \mathbf{4} \vee \ldots \leftarrow 4 \vee \ldots \leftarrow \ldots \text{(SA)}$$

O

- asserts prejacent
- negates pre-exh'ed non-ent'd DA

In episodic contexts:

- contradiction

Across an intervening modal:

- FC effect ✓
  - For necessity modal:

    Just SgDA: $\Box_S \neg n$ ok
    Just NonSgDA: $\Box_S n$ ok
    all DA: total FC

SMNs (but not CMNs) require all DA!

- total FC ✓

(30) Jo didn't solve # at most 2 problems.

$$0 \quad 1 \quad 2 \qquad \text{(DA)}$$
$$0\lor 1 \quad 0\lor 2 \quad 1\lor 2$$
$$\uparrow$$
$$0 \leftarrow 0\lor 1 \leftarrow \mathbf{0}\lor\mathbf{1}\lor\mathbf{2} \leftarrow \cdots\lor 3 \leftarrow \ldots \text{(SA)}$$

(31) Jo didn't solve # at least 3 problems.

$$3 \quad 4 \quad 5 \quad \ldots \qquad \text{(DA)}$$
$$3\lor 4 \quad 3\lor 4\lor 5 \quad 4\lor 7\lor 8\lor 10 \quad \ldots$$
$$\uparrow$$
$$\ldots \rightarrow 2\lor\ldots \rightarrow \mathbf{3}\lor\mathbf{4}\lor\ldots \rightarrow 4\lor\ldots \rightarrow \ldots \text{(SA)}$$

O

- asserts prejacent +presup
- negates non-ent'd pre-exh'd DA

Across *not*:

- non-ent'd ExhDA already excluded
  - O_{ExhDA} vacuous!

Across *if/every/only*:

- due to pos. presupposition:
  - $O_{ExhDA} \Rightarrow$ FC effect

SMNs (but not CMNs) need O_{DA} to lead to proper strengthening (PS)!

- POL1 ✓

(32) Jo solved at most 2 problems.

$$0 \quad 1 \quad 2 \qquad \text{(DA)}$$
$$0 \vee 1 \quad 0 \vee 2 \quad 1 \vee 2$$
$$\downarrow$$
$$0 \rightarrow 0 \vee 1 \rightarrow \mathbf{0} \vee \mathbf{1} \vee \mathbf{2} \rightarrow \cdots \vee 3 \rightarrow \ldots \text{(SA)}$$

(33) Jo solved at least 3 problems.

$$3 \quad 4 \quad 5 \quad \ldots \qquad \text{(DA)}$$
$$3 \vee 4 \quad 3 \vee 4 \vee 5 \quad 4 \vee 7 \vee 8 \vee 10 \quad \ldots$$
$$\downarrow$$
$$\ldots \leftarrow 2 \vee \ldots \leftarrow \mathbf{3} \vee \mathbf{4} \vee \ldots \leftarrow 4 \vee \ldots \leftarrow \ldots \text{(SA)}$$

O

- asserts the prejacent
- negates the non-entailed SA

In some contexts, for granularity = 1:

- SI ⇒'exactly' meanings
- these clash with FC

Clash between FC (DA-implic's) and SI (SA-implic's) solved by SA-pruning!

- Just the desired SI ✓

Taking stock

alternatives	exh'ivity op	extra ingredients	phenomenon
pre-exh'd, non-ent'd DA	O(nly)	$\Box_S$, ± DA-pruning	FC ✓
pre-exh'd, non-ent'd DA	O(nly)	presuppositions, ± PS	POL1 ✓
non-ent'd SA	O(nly)	SA-pruning, granularity	SI ✓

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Conclusion and outlook

(34) If Jo solved ✓**at least 3** / # **at most 3** problems, she passed.

(35) If Jo solved # **at least 3** / ✓**at most 3** problems, she failed.

(36) If Jo made # **at least 3** / ✓**at most 3** mistakes, she passed.

(37) If Jo made ? **at least 3** / # **at most 3** mistakes, she failed.

(38) If Jo didn't solve # **at least 3** / ? **at most 3** problems, she passed.

(39) If Jo didn't solve ✓**at least 3** / # **at most 3** problems, she failed.

(40) If Jo didn't make ? **at least 3** / # **at most 3** mistakes, she passed.

(41) If Jo didn't make # **at least 3** / ? **at most 3** mistakes, she failed.

Same effects for CMNs, though weaker.

Most of the literature ignores all of these patterns. However

- Discussion: [Cohen and Krifka, 2014], also citing [Kay, 1992] and [Nilsen, 2007].
- Experimental validation: [Mihoc and Davidson, 2017] (to appendix »)

[Cohen and Krifka, 2014] discuss both POL1 and POL2.

- Main argument: They go back to two different meanings of SMs:
  - one non-evaluative which is always bad in DE environments
  - one evaluative which can be fine in DE environments
    - presupposition: the property that the SMN combines with is in some sense a good thing
- Problems: For both the account of POL1 and the account of POL2:
  - evaluative meanings are still bad under negation, no matter their valence:

    (42) This hotel isn't # at least centrally located.

    (43) This hotel isn't # at least far away.

  - evaluative meanings sensitive to polarity of the modifier also, as we have seen

POL2 is reminiscent of effects reported by [Crnič, 2011] (and refs) for minimizers:

(44) Everyone that **lifted a finger** to help was rewarded / # wearing blue jeans.

In an alternatives-and-exhaustification framework minimizers have been analyzed in terms of scalar alternatives and exhaustification with a silent exhaustivity operator E(ven) [Crnič, 2011], [Chierchia, 2013].

Even presupposes that its prejacent is the least likely among a set of scalar alternatives.

(45) John read even # **one** book.
$\#$ read one $\prec$ read two

(46) Even if John read ✓**one** book, he will (still) pass the exam.
✓read one $\rightarrow$ pass $\prec$ read two $\rightarrow$ pass

In some cases this presupposition is impossible to satisfy, yet the result is still fine.

(47) Even if John read ✓**all** of the books, he will (still) fail the exam.
$\#$ read all $\rightarrow$ fail $\prec$ read some $\rightarrow$ fail

Suggestion: The SA may be interpreted exhaustively. Likelihood assessed not based on logical strength but rather based on contextual plausibility.

(48) read O_{SA}(all) (= all) $\rightarrow$ fail $\prec_c$ read O_{SA}(some) (= some but not all) $\rightarrow$ fail ✓

We will make the same assumptions about silent E(ven) also.

(49) Jo solved at most 2 problems.

$$0 \quad 1 \quad 2 \qquad \text{(DA)}$$

$$0 \vee 1 \quad 0 \vee 2 \quad 1 \vee 2$$

$$\downarrow$$

$$0 \rightarrow 0 \vee 1 \rightarrow \mathbf{0} \vee \mathbf{1} \vee \mathbf{2} \rightarrow \cdots \vee 3 \rightarrow \ldots \text{(SA)}$$

(50) Jo solved at least 3 problems.

$$3 \quad 4 \quad 5 \quad \ldots \qquad \text{(DA)}$$

$$3 \vee 4 \quad 3 \vee 4 \vee 5 \quad 4 \vee 7 \vee 8 \vee 10 \quad \ldots$$

$$\downarrow$$

$$\ldots \leftarrow 2 \vee \ldots \leftarrow \mathbf{3} \vee \mathbf{4} \vee \ldots \leftarrow 4 \vee \ldots \leftarrow \ldots \text{(SA)}$$

- [Crnič, 2011] discusses cases where the item is end-of-scale.
- But our SMNs are usually not.
- They have both stronger & weaker SA.
- Which SA does E consider?

If the non-entailed SA, the presupposition cannot be satisfied (based on logic). ✗

(51) If Jo solved ✓**at least 3** problems, she passed.
at least 3 solutions → pass $\prec$ at least 2 solutions → pass

If the pre-exh'ed non-entailed SA, wrong predictions (based on context). ✗

(52) If Jo solved ✓**at least 3** problems, she passed.
exactly 3 solutions → pass $\prec_c$ exactly 2 solutions → pass

If the entailed SA, the presup. is trivially satisfied (based on logic). Can't capture contrasts. ✗

(53) If Jo solved ✓**at least 3** problems, she passed.
✓at least 3 solutions → pass $\prec$ at least 4 solutions → pass

(54) If Jo solved # **at least 3** problems, she failed.
✓at least 3 solutions → fail $\prec$ at least 4 solutions → fail

If the pre'exh'ed entailed SA, just the right predictions (based on context). ✓

(55) If Jo solved ✓**at least 3** problems, she passed.
✓exactly 3 solutions → pass $\prec_c$ exactly 4 solutions → pass

(56) If Jo solved # **at least 3** problems, she failed.
exactly 3 solutions → fail $\prec_c$ exactly 4 solutions → fail

E pitches the prejacent up against its *entailed* SA, considered in *pre-exhaustified* form.

correct predictions

(57) If Jo solved ✓**at least 3** problems, she passed.
✓exactly 3 solutions → pass $\prec_c$ exactly 4 solutions → pass

(58) If Jo solved # **at least 3** problems, she failed.
exactly 3 solutions → fail $\prec_c$ exactly 4 solutions → fail

(59) If Jo solved # **at most 3** problems, she passed.
exactly 3 solutions → pass $\prec_c$ exactly 2 solutions → pass

(60) If Jo solved ✓**at most 3** problems, she failed.
✓exactly 3 solutions → fail $\prec_c$ exactly 2 solutions → fail

correct predictions

(61) If Jo made # **at least 3** mistakes, she passed.
exactly 3 mistakes → pass $\prec_c$ exactly 4 mistakes → pass

(62) If Jo made ? **at least 3** mistakes, she failed.
✓exactly 3 mistakes → fail $\prec_c$ exactly 4 mistakes → fail

(63) If Jo made ✓**at most 3** mistakes, she passed.
✓exactly 3 mistakes → pass $\prec_c$ exactly 2 mistakes → pass

(64) If Jo made # **at most 3** mistakes, she failed.
exactly 3 mistakes → fail $\prec_c$ exactly 2 mistakes → fail

didn't solve problems → pass/fail

(65) If Jo didn't solve # **at least 3** problems, she passed.
exactly 3 solutions → pass $\prec_c$ exactly 2 solutions → pass

(66) If Jo didn't solve ✓**at least 3** problems, she failed.
✓exactly 3 solutions → fail $\prec_c$ exactly 2 solutions → fail

(67) If Jo didn't solve ? **at most 3** problems, she passed.
✓exactly 3 solutions → pass $\prec_c$ exactly 4 solutions → pass

(68) If Jo didn't solve # **at most 3** problems, she failed.
exactly 3 solutions → fail $\prec_c$ exactly 4 solutions → fail

(69) If Jo didn't make ? **at least 3** mistakes, she passed.
✓exactly 3 mistakes → pass $\prec_c$ exactly 2 mistakes → pass

(70) If Jo didn't make # **at least 3** mistakes, she failed.
exactly 3 mistakes → fail $\prec_c$ exactly 2 mistakes → fail

(71) If Jo didn't make # **at most 3** mistakes, she passed.
exactly 3 mistakes → pass $\prec_c$ exactly 4 mistakes → pass

(72) If Jo didn't make ? **at most 3** mistakes, she failed.
✓exactly 3 mistakes → fail $\prec_c$ exactly 4 mistakes → fail

further welcome predictions

(73) Jo solved at least 3 problems. ☺
exactly 3 $\prec_c$ exactly 2
'That's many solutions.'

(74) Jo made at least 3 mistakes. ☹
exactly 3 $\prec_c$ exactly 2
'That's many mistakes.'

(75) Jo bought at least 3 phones. —
exactly 3 $\prec_c$ exactly 2
'That's many phones.'

(76) Jo solved at most 3 problems. ☹
exactly 3 $\prec_c$ exactly 4
'That's few solutions.'

(77) Jo made at most 3 mistakes. ☺
exactly 3 $\prec_c$ exactly 4
'That's few mistakes.'

(78) Jo bought at most 3 phones. —
exactly 3 $\prec_c$ exactly 4
'That's few phones.'

POL2+FC+SI the key to an old puzzle with possibility modals: *at most*

(79) Jo may drink ✓at most 3 beers.

$$0 \quad 1 \quad 2 \quad 3 \qquad\qquad \text{(DA)}$$

$$0 \vee 1 \quad 0 \vee 2 \quad 1 \vee 2 \quad 0 \vee 1 \vee 3 \ldots$$

$$\downarrow$$

$$\ldots \leftarrow \cdots \vee 2 \leftarrow \mathbf{0} \vee \mathbf{1} \vee \mathbf{2} \vee \mathbf{3} \leftarrow \cdots \vee 4 \leftarrow \ldots \text{(SA)}$$

O_{ExhDA} ends up reversing the scale.

This affects both O_{SA} and E_{SA}

(80) Jo may drink ✓**at most** 3 beers.

$E_{ExhSA}, O_{SA}(O_{ExhDA}(\Diamond(\cdots \vee 2 \vee 3)))$
$= E_{ExhSA}, O_{SA}(\Diamond 0 \wedge \Diamond 1 \wedge \Diamond 2 \wedge \Diamond 3)$

$O_{SA} : (\cdots \wedge \Diamond 3) \wedge \neg(\cdots \wedge \Diamond 4)$
'No more.'
the upper bound of *at most* under $\Diamond$!

$E_{ExhSA} : O_{SA}(\cdots \wedge \Diamond 3) \prec_c O_{SA}(\cdots \wedge \Diamond 2)$
$= \Diamond 3 \prec_c \Diamond 2$
✓exactly 3 $\prec_c$ exactly 2
'That's many.'
fits with typical assumptions

POL2+FC+SI the key to an old puzzle with possibility modals: *at least*

(81) Jo may drink # at least 3 beers.

$$3 \quad 4 \quad 5 \quad \dots \qquad \text{(DA)}$$

$$3 \vee 4 \quad 3 \vee 4 \vee 5 \quad 4 \vee 7 \vee 8 \vee 10 \quad \dots$$

$$\downarrow$$

$$\dots \rightarrow 2 \vee \dots \rightarrow \mathbf{3} \vee \mathbf{4} \vee \dots \rightarrow 4 \vee \dots \rightarrow \dots \text{(SA)}$$

O_{ExhDA} ends up reversing the scale.

This affects both O_{SA} and E_{SA}

(82) Jo may drink # **at least** 3 beers.

$E_{ExhSA}, O_{SA}(O_{ExhDA}(\Diamond(3 \vee \dots)))$

$= E_{ExhSA}, O_{SA}(\Diamond 3 \wedge \Diamond 4 \wedge \dots)$

$O_{SA} : (\Diamond 3 \wedge \dots) \wedge \neg(\Diamond 2 \wedge \dots)$

'No less.'

odd lower bound

$E_{ExhSA} : O_{SA}(\Diamond 3 \wedge \dots) \prec_c O_{SA}(\Diamond 4 \wedge \dots)$

$= \Diamond 3 \prec_c \Diamond 4$

✓exactly 3 $\prec_c$ exactly 4

'That' few.'

doesn't fit with typical assumptions

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Conclusion

SMNs (and CMNs) exhibit a bewildering array of effects, often in interraction.

We can make sense of them by studying their alternatives and their use.

SMNs emerge as items that want all their alternatives to contribute to strengthening, and recruit both O and E, both their ent'd and non-ent'd alternatives to attain that.

alternatives	exh'ivity op	extra ingredients	phenomenon
pre-exh'd, non-ent'd DA	O(nly)	$\Box_S$, ± DA-pruning	FC ✓
pre-exh'd, non-ent'd DA	O(nly)	non-tc content, ± PS	POL1 ✓
non-ent'd SA	O(nly)	SA-pruning, granularity	SI ✓
pre-exh'd, *entailed* SA	E(ven)	contextual assumptions	POL2 ✓

- A reviewer points out that with an overt *even* the opposite patterns obtain:

(83) If Jo solved ✓**at least 3** problems, she passed.

(84) Even if Jo solved # **at least 3** problems, she passed.

(85) If Jo solved # **at most 3** problems, she passed.

(86) Even if Jo solved ✓**at most 3** problems, she passed.

- [Horn, 1972] notes similar contrasts between what we now take to be O and *only*:

(87) 60 % if not ✓more / # less of the electorate will be fooled.

(88) Only 60 % if not # more / ✓less of the electorate will be fooled.

- I believe this has to do with differences between the covert and the overt w.r.t. what is asserted and what is presupposed.

Outlook

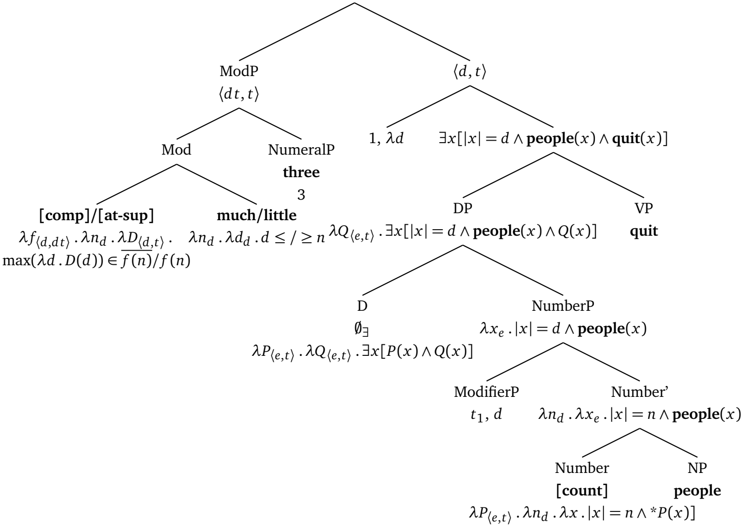
What is the connection between E and O?

- We've seen some of their interaction but we want to understand it much better.

Thank you!

Appendix: The syntax and semantics of CMNs and SMNs (to main »)

$$[\![\text{More/less than three / at most/least three people quit}]\!]$$
$$= 1 \textit{ iff } \max(\lambda d\,.\,\exists x[|x| = d \wedge \textbf{people}(x) \wedge \textbf{quit}(x)]) \in \underbrace{[\![\text{much/little}]\!]\,(3)}_{\{4,\ldots\}/\{\ldots,2\}} / \underbrace{\overline{[\![\text{much/little}]\!]\,(3)}}_{\{\ldots,3\}/\{3,\ldots\}}$$



The syntactic assumptions about [count] being the head of a functional projection NumberP intermediary between the DP and the NP and the bare numeral being a phrasal projection NumeralP merged in the specifier of NumberP are as in [Zabbal, 2005], [Scontras, 2013], and references therein, though here I extend this assumption to modified numerals and their phrasal projection (what I call 'ModifierP').

Note: In NumberP, by replacing ModifierP with NumeralP, one also gets the syntax and semantics of bare numerals (BNs). (I assume that a bare numeral denotes a simple degree; its predicative meaning is derived, for example, via typeshifting, as in [Buccola and Spector, 2016].)

(89) $O_{ExhDA}\Diamond(0 \vee 1)$

$$= \Diamond(0 \vee 1) \wedge \neg \underbrace{\underbrace{O\Diamond 0}_{\Diamond 0 \wedge \neg \Diamond 1}}_{\Diamond 0 \to \Diamond 1} \wedge \neg \underbrace{\underbrace{O\Diamond 1}_{\Diamond 1 \wedge \neg \Diamond 0}}_{\Diamond 1 \to \Diamond 0}$$

$$= \Diamond 0 \wedge \Diamond 1$$

(90) $O_{ExhDA}\Box(0 \vee 1)$

$$= \Box(0 \vee 1) \wedge \neg \underbrace{\underbrace{O\Box 0}_{\Box 0 \wedge \neg \Box 1}}_{\Box 0 \to \Box 1} \wedge \neg \underbrace{\underbrace{O\Box 1}_{\Box 1 \wedge \neg \Box 0}}_{\Box 1 \to \Box 0}$$

$$= \underbrace{\Box(0 \vee 1) \wedge \neg \Box 0 \wedge \neg \Box 1}_{\to \Diamond 0 \wedge \Diamond 1}$$

(91) $O_{ExhDA}(0 \vee 1)$

$$= (0 \vee 1) \wedge \neg \underbrace{\underbrace{O0}_{0 \wedge \neg 1}}_{0 \to 1} \wedge \neg \underbrace{\underbrace{O1}_{1 \wedge \neg 0}}_{1 \to 0}$$

$$= (0 \vee 1) \wedge \neg 0 \wedge \neg 1$$

$$= \bot$$

(92) $O_{ExhDA}\Box_S(0 \vee 1)$

$$= \Box_S(0 \vee 1) \wedge \neg \underbrace{\underbrace{O\Box_S 0}_{\Box_S 0 \wedge \neg \Box_S 1}}_{\Box_S 0 \to \Box_S 1} \wedge \neg \underbrace{\underbrace{O\Box_S 1}_{\Box_S 1 \wedge \neg \Box_S 0}}_{\Box_S 1 \to \Box_S 0}$$

$$= \underbrace{\Box_S(0 \vee 1) \wedge \neg \Box_S 0 \wedge \neg \Box_S 1}_{\to \Diamond_S 0 \wedge \Diamond_S 1}$$

Appendix: POL1: computations

(to main »)

(93) $O_{\text{ExhDA}}(\neg(0 \vee 1))$
$= \neg(0 \vee 1) \wedge$
$\neg \underbrace{(\neg 0 \wedge \neg\neg 1)}_{\text{already excluded}} \wedge \neg \underbrace{(\neg 1 \wedge \neg\neg 0)}_{\text{already excluded}}$

$\Rightarrow O_{\text{ExhDA}}$ vacuous

(94) $O^{S}_{\text{ExhDA}} \forall w[(0 \vee 1)_w \to W_w]$
$= \forall w[(0 \vee 1)_w \to W_w] \wedge \exists w[(0 \vee 1)_w] \wedge$
$(\cdots \wedge \exists w[0_w]) \to (\cdots \wedge \exists w[1_w]) \wedge$
$(\cdots \wedge \exists w[1_w]) \to (\cdots \wedge \exists w[0_w])$

$\Rightarrow O^{S}_{\text{ExhDA}}$ (which takes into account the existential presupposition of conditionals) leads to FC effect

Appendix: SI: computations

(to main »)

As in [Horn, 1972]. 'Exactly' results of SI actually clash with FC. Assumption: Clash fixed by removing the offending SA.

(95) Jo called at most two people. $\not\rightsquigarrow$ 'exactly 2'

$O_{ExhDA}(\Box_S O_{SA}(0 \vee 1 \vee 2))$

a. $\Box_S O_{SA}(0 \vee 1 \vee 2) \wedge$

b. $\neg O \Box_S 0 \wedge \neg O \Box_S 1 \wedge \neg O \Box_S 2 \wedge \neg O \Box_S (0 \vee 1) \wedge \neg O \Box_S (1 \vee 2) \wedge \neg O \Box_S (0 \vee 2)$

$$= \underbrace{\underbrace{\overbrace{\Box_S((0 \vee 1 \vee 2) \wedge \neg(0 \vee 1))}^{(a)}}_{= \Box_S 2} \wedge \overbrace{\neg \Box_S 0 \wedge \neg \Box_S 1 \wedge \neg \Box_S 2}^{(b)}}_{\bot}$$

($\bot$ resolved by default SA-pruning)

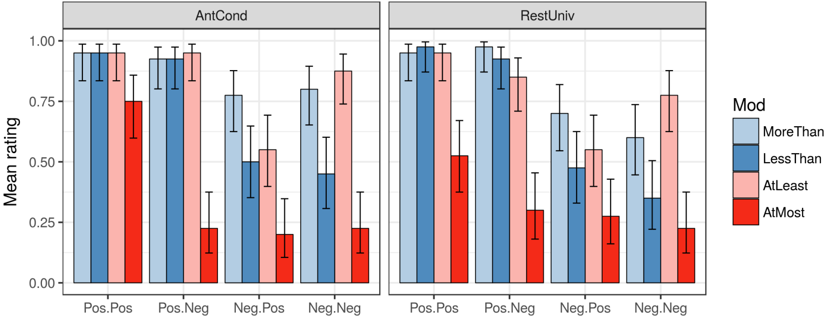
'Exactly' results of SI for scalar under negation are actually not generated if we assume ALT(not at most 2) $= \{\ldots$, not am1, not am2, $\ldots$, am1, am2, $\ldots\}$.

Appendix: Preliminary experimental evidence for POL2 (to main »)

Meowth remembers:

Meowth says: If you don't have at least 3 hearts, you lose.

Do you think the other players will understand what he said?



Context: Player partially ignorant of neighbor's hand setting up rules to affect neighbor's hand. **Item summary:** (AntCond) If you have/don't have [Mod] 3 [suit], you win/lose. (RestUniv) Everyone who has/doesn't have [Mod] 3 [suit] wins/loses.

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