(Scalarity,) ignorance and positive polarity

in

indefinites, disjunction, and numerals

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Outline

Preamble

Q1: Are the facts parallel?

Q2: What is the parallel account?

Q3: Why should we care?

It is a truth universally acknowledged that
a single man in possession of a good fortune
must be in want of
a wife.

It is a truth universally acknowledged that
~~a single man in possession of a good fortune~~ parallel facts
must be in want of
a wife.

It is a truth universally acknowledged that
~~a single man in possession of a good fortune~~ parallel facts
must be in want of
~~a wife~~ a parallel account.

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Q2: What is the parallel account?

In this talk I tackle these questions for the ignorance and positive polarity of numerals, disjunction, and indefinites:
3 (BNs) *more/less than 3* (CMNs) *at least/most 3* (SMNs) *Alice, Bob, or Cindy* *some student*

It is a truth universally acknowledged that
~~a single man in possession of a good fortune~~ parallel facts
must be in want of
~~a wife~~ a parallel account.

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Q2: What is the parallel account?

In this talk I tackle these questions for the ignorance and positive polarity of numerals, disjunction, and indefinites:
3 (BNs) *more/less than 3* (CMNs) *at least/most 3* (SMNs) *Alice, Bob, or Cindy* *some student*

Q3: Why should we care?

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Q1: Are the facts parallel?

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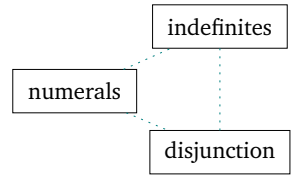
Numerals, disjunction, indefinites

Horn (1972)

(1) Jo called 3 students / A or B / some student.

a. Jo called 4/5/... students / A and B / every student.

b. ¬ Jo called 4/5/... students / A and B / every student.
⇒ exactly 3 / A xor B / some-but-not-every

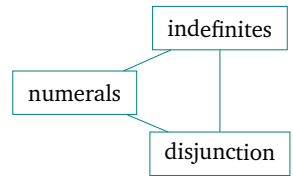


(2) Lexical scales + Gricean reasoning

a. ⟨..., two, three, four,...⟩; three ⇝ not four

b. ⟨or, and⟩; or ⇝ not and

c. ⟨some, every⟩; some ⇝ not every



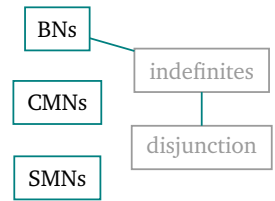
Exit modified numerals

Krifka (1999)

(3) Jo called more than 2 / less than 4 / at least 3 / at most 3 students.

a. more than 3 / less than 3 / at least 4 / at most 2

b. ¬ # more than 3 / # less than 3 / # at least 4 / # at most 2
⇒ # exactly 3 / # exactly 3 / # exactly 3 / # exactly 3



Exit superlative-modified numerals

Geurts and Nouwen (2007), Cohen and Krifka (2014), Mihoc and Davidson (2021)

(4) Jo called **3** students. So, she called ✓**more than 2** / # **at least 3**.

(5) a. Jo did**n't** call ✓**more than 2** / # **at least 3** students.

b. **No**body called ✓**more than 2** / # **at least 3** people.

c. Jo managed **without** calling ✓**more than 2** / # **at least 3** people.

d. **Few** of the participants called ✓**more than 2** / # **at least 3** people.

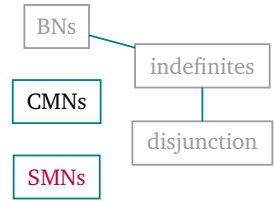
e. Jo **rarely** called ✓**more than 2** / # **at least 3** people.

f. **If** Jo called ✓**more than 2** / ✓**at least 3** people, she won.

g. **Every**one who called ✓**more than 2** / ✓**at least 3** people won.

h. Tim does**n't** know that Jo called ✓**more than 2** / ✓**at least 3** people.

i. **Only** kids aged ✓**more than 2** / ✓**at least 3** can attend.

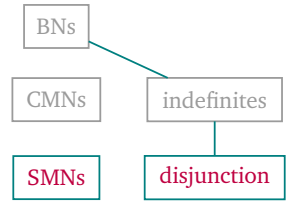


Re-enter disjunction

Büring (2008); Mihoc (2020, 2021), building on Strawson (1952), Grice (1989), Rips (1994), Chierchia (2013), Spector (2014), Nicolae (2017) a.o.

(6) Jo called Alice. So, she called # A, B, **or** C / # A, B, **ou** C.

(7) a. Jo did**n't** call ✓A, B, **or** C / # A, B, **ou** C.
b. **No**body called ✓A, B, **or** C / # A, B, **ou** C.
c. Jo managed **without** calling ✓A, B, **or** C / # A, B, **ou** C.
d. **Few** of the participants called ✓A, B, **or** C / ✓A, B, **ou** C.
e. Jo **rarely** called ✓A, B, **or** C / ✓A, B, **ou** C.
f. **If** Jo called ✓A, B, **or** C / ✓A, B, **ou** C, she won.
g. **Every**one who called ✓A, B, **or** C / ✓A, B, **ou** C won.



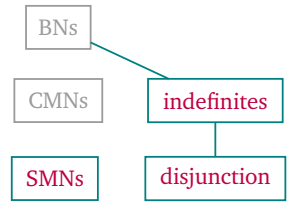
Re-enter indefinites

Nouwen (2015); Mihoc (2021), building on Strawson (1974), Becker (1999), Chierchia (2013), Alonso-Ovalle and Menéndez-Benito (2015), Kratzer and Shimoyama (2017) a.o.

(8) a. Jo called **Alice**. So, she called # **un** student **oarecare** / # **irgendein** student / ✓**some** student.
b. Jo called # **un** student **oarecare** / ✓**irgendein** student / ✓**some** student, but **not Alice**.

(9) **Some** cabinet minister has been shot.
⇝ Speaker ignorance or indifference

(10) a. Jo di**dn't** call # **un** student **oarecare** / # **irgendein** student / # **some** student.
b. **No**body called # **un** student **oarecare** / ✓**irgendein** student / # **some** student.
c. Jo managed **without** calling # **un** student **oarecare** / ✓**irgendein** student / # **some** student.
d. **Few** of the participants called # **un** student **oarecare** / ✓**irgendein** student / ✓**some** student.
e. Jo **rarely** called # **un** student **oarecare** / ✓**irgendein** student / ✓**some** student.
f. **If** Jo called ✓**un** student **oarecare** / ✓**irgendein** student / ✓**some** student, she won.
g. **Every**one who called ✓**un** student **oarecare** / ✓**irgendein** student / ✓**some** student won.



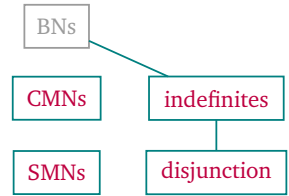
Re-enter comparative-modified numerals

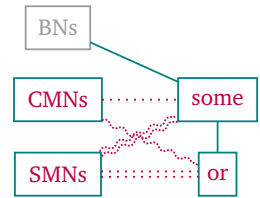
Mihoc (2020, 2021), building also on findings from indefinites, Mayr and Meyer (2014), Westera and Brasoveanu (2014), Cremers et al. (2017)

(11) a.

b. Jo called ✓**more than 2** / # **at least 3** students, but **not 5**.

(12) Jo called **more than 2** students.
⇝ Speaker ignorance or indifference





Re-enter bare numerals, modified numerals

Mihoc (2021), building especially on Mayr (2013), Spector (2013, 2014)

(13) a. Jo called 3 / more than 2 / less than 4 / at least 3 / at most 3 students.
⇝ ¬ ✓4 / # more than 3 / # less than 3 / # at least 4 / # at most 2
⇒ ✓exactly 3 / # exactly 3 / # exactly 3 / # exactly 3 / # exactly 3

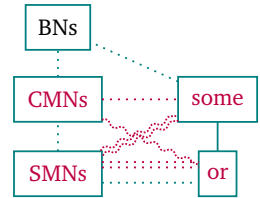
b. Jo didn't call 3 / more than 2 / less than 4 / # at least 3 / # at most 3 students.
⇝ ¬ not # 2 / # more than 1 / # less than 5 / # at least 2 / # at most 4
⇒ # exactly 2 / # exactly 4 / # exactly 2 / # exactly / # exactly 4

c. Everyone called 3 / more than 2 / less than 4 / at least 3 / at most 3 students.
⇝ ¬ everyone ✓4 / ✓more than 3 / ✓less than 3 / ✓at least 4 / ✓at most 2

d. If Jo called 3 / more than 2 / less than 4 / at least 3 / at most 3 students, she won.
⇝ ¬ if ✓2 / ✓more than 1 / ✓less than 5 / ✓at least 2 / ✓at most 4

e. Jo called 3 / more than 2 / less than 4 / at least 3 / at most 3 students.
⇝ ¬ ✓4 / ✓more than 4 / ✓less than 2 / ✓at least 5 / ✓at most 1

f. Jo didn't call 3 / more than 2 / less than 4 / # at least 3 / # at most 3 students.
⇝ ¬ not ✓1 / ✓more than 0 / ✓less than 6 / ✓at least 1 / ✓at most 5



		Ignorance / Other modal variation effects?			Positive polarity?			Scalar implicatures?
Category	**Item**	**Variation effect**	**Positive specificity**	**Negative specificity**	**Plain DE**	**DE + pos. presup.**	**DE + pos. implic.**	
Indefinites	**un qualsiasi/qualunque NP (Italian)**	yes	no	no	yes	yes	yes	yes
	un NP qualsiasi/qualunque (Italian)	yes	no	no	no	yes	no	yes
	un NP oarecare (Romanian)	yes	no	no	no	yes	no	yes
	un qualche NP (Italian)	yes	no	yes	no	yes	?	yes
	algún (Spanish)	yes	no	yes	no	yes	yes	yes
	irgendein (German)	yes	no	yes	yes	yes	?	yes
	some (English)	yes	yes	yes	no	yes	yes	yes
Disjunction	**ou**	yes	no	no	no	yes	yes	yes
	or	yes	no	no	yes	yes	yes	yes
Numerals	**BNs (e.g., three)**	no*	NA	NA	yes	yes	yes	yes*
	CMNs (e.g., more/less than three)	yes	yes	yes	yes	yes	yes	yes*
	SMNs (e.g., at least/most three)	yes	no	no	no	yes	no	yes*

Q1: Are the facts parallel?

Yes, though in a much richer sense than usually acknowledged.

In all of numerals, disjunction and indefinites we find:

- a modal variation effect in seemingly episodic contexts:
  - ± compatibility with specificity (negative, positive, negative & positive)
  - ± compatibility with DE environments (plain, +positive implicature, +positive presupposition)
- Horn-style scalar implicatures

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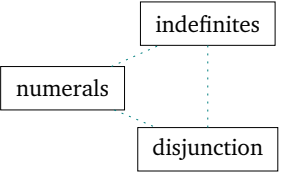
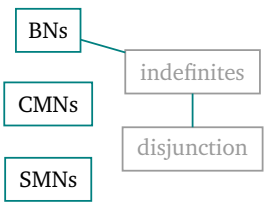
Q3: Why should we care?



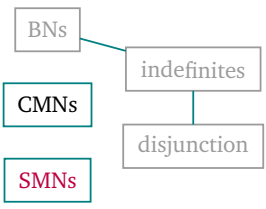
Horn (1972), Chierchia et al. (2012), a.o.:

- Numerals, disjunction, and indefinites naturally activate scalar alternatives.
- Factoring these in yields scalar implicatures.



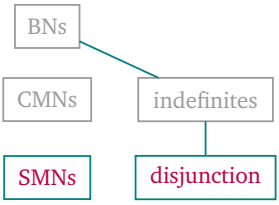
Krifka (1999), Fox and Hackl (2006), Mayr (2013), Coppock and Brochhagen (2013), Kennedy (2015), Schwarz (2016), etc.:

- MNs do not give rise to Horn scalar implicatures.



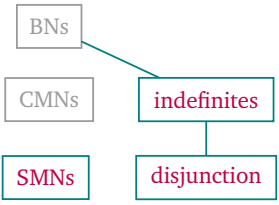
Krifka (1999), Geurts and Nouwen (2007), Nouwen (2010), Cohen and Krifka (2014):

- SMNs are special.



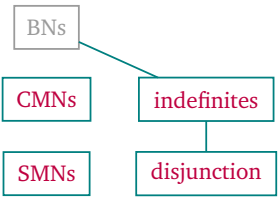
Büring (2008), Coppock and Brochhagen (2013), Kennedy (2015), Schwarz (2016), Spector (2015), Nicolae (2017), Mihoc (2020, 2021):

- SMNs are disjunction-like.
- They activate disjunctive alternatives.
- Factoring these in yields total ignorance and positive polarity.



Nouwen (2015), Alonso-Ovalle and Menéndez-Benito (2010), Chierchia (2013), Mihoc (2020, 2021):

- SMNs and disjunction are indefinite-like.
- They activate subdomain alternatives.
- Factoring these in yields ignorance ± compatibility with negative and/or positive specificity and ± positive polarity.



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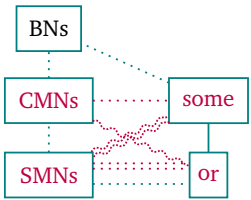

Mihoc (2020, 2021):

- Numerals, disjunction, and indefinites are item-with-domain- and item-with-scale-like.
- They naturally activate scalar and, except for BNs, also subdomain alternatives.
- Factoring these in (in very specific ways) yields ignorance ± compatibility with negative and/or positive specificity and ± polarity sensitivity, and Horn-style scalar implicatures.
- In numerals, in certain contexts, due to the nature of the domain, ignorance and scalar implicatures clash, hence the occasional scalar implicature gaps.

Concrete illustration: Truth conditions and alternatives

(14) Jo called Alice or Bob.
Jo called some student$_{\{\text{Alice, Bob}\}}$.

$$\begin{array}{ccc} a \quad b & & \text{(DA)} \\ \downarrow & & \\ \mathbf{a \vee b} & \leftarrow \quad a \wedge b & \text{(SA)} \end{array}$$

(15) Jo called less than 2 people.
Jo called at most 1 person.

$$\begin{array}{ccc} 0 \quad 1 & & \text{(DA)} \\ \downarrow & & \\ 0 \rightarrow \mathbf{0 \vee 1} \rightarrow 0 \vee 1 \vee 2 \rightarrow \ldots & & \text{(SA)} \end{array}$$

(16) Jo called Alice, Bob, or Cindy.
Jo called some student$_{\{\text{Alice, Bob, Cindy}\}}$.

$$\begin{array}{ccc} a \quad b \quad c & & \text{(DA)} \\ a \vee b \quad a \vee c \quad b \vee c & & \\ \downarrow & & \\ \mathbf{a \vee b \vee c} \leftarrow a \wedge b, \ldots \leftarrow a \wedge b \wedge c & & \text{(SA)} \end{array}$$

(17) Jo called less than 3 people.
Jo called at most 2 people.

$$\begin{array}{ccc} 0 \quad 1 \quad 2 & & \text{(DA)} \\ 0 \vee 1 \quad 0 \vee 2 \quad 1 \vee 2 & & \\ \downarrow & & \\ 0 \rightarrow 0 \vee 1 \rightarrow \mathbf{0 \vee 1 \vee 2} \rightarrow \cdots \vee 3 \rightarrow \ldots & & \text{(SA)} \end{array}$$

Concrete illustration: Implicature calculation mechanism

Contradiction-based O(nly). Pre-exhaustification (with IE) relative to DA of same size (or smaller). (* = crucial only for computations with ◇)

(18) $\mathrm{O}_{\mathrm{ExhDA}}(a \vee b) =$
$(a \vee b) \wedge \neg \underbrace{\mathrm{O}a}_{\substack{a \wedge \neg b \\ a \to b}} \wedge \neg \underbrace{\mathrm{O}b}_{\substack{b \wedge \neg a \\ b \to a}}, = a \wedge b$

(19) $\mathrm{O}_{\mathrm{ExhDA}}(0 \vee 1) =$
$(0 \vee 1) \wedge \neg \underbrace{\mathrm{O}0}_{\substack{0 \wedge \neg 1 \\ 0 \to 1}} \wedge \neg \underbrace{\mathrm{O}1}_{\substack{1 \wedge \neg 0 \\ 1 \to 0}}, = \bot$

(20) $\mathrm{O}_{\mathrm{SA}}(a \vee b) = (a \vee b) \wedge \neg(a \wedge b)$

(21) $\mathrm{O}_{\mathrm{SA}}(0 \vee 1) = (0 \vee 1) \wedge \neg 0, = 1$

Concrete illustration: Ignorance [👉]

(22) $\mathrm{O}_{\mathrm{ExhDA}}\Diamond(a \vee b)$
$= \Diamond(a \vee b) \wedge \neg \underbrace{\mathrm{O}\Diamond a}_{\substack{\Diamond a \wedge \neg \Diamond b \\ \Diamond a \rightarrow \Diamond b}} \wedge \neg \underbrace{\mathrm{O}\Diamond b}_{\substack{\Diamond b \wedge \neg \Diamond a \\ \Diamond b \rightarrow \Diamond a}}$

(23) $\mathrm{O}_{\mathrm{ExhDA}}\Diamond(0 \vee 1)$
$= \Diamond(0 \vee 1) \wedge \neg \underbrace{\mathrm{O}\Diamond 0}_{\substack{\Diamond 0 \wedge \neg \Diamond 1 \\ \Diamond 0 \rightarrow \Diamond 1}} \wedge \neg \underbrace{\mathrm{O}\Diamond 1}_{\substack{\Diamond 1 \wedge \neg \Diamond 0 \\ \Diamond 1 \rightarrow \Diamond 0}}$

(24) $\mathrm{O}_{\mathrm{ExhDA}}\Box(a \vee b)$
$= \Box(a \vee b) \wedge \neg \underbrace{\mathrm{O}\Box a}_{\substack{\Box a \wedge \neg \Box b \\ \Box a \rightarrow \Box b}} \wedge \neg \underbrace{\mathrm{O}\Box b}_{\substack{\Box b \wedge \neg \Box a \\ \Box b \rightarrow \Box a}}$

(25) $\mathrm{O}_{\mathrm{ExhDA}}\Box(0 \vee 1)$
$= \Box(0 \vee 1) \wedge \neg \underbrace{\mathrm{O}\Box 0}_{\substack{\Box 0 \wedge \neg \Box 1 \\ \Box 0 \rightarrow \Box 1}} \wedge \neg \underbrace{\mathrm{O}\Box 1}_{\substack{\Box 1 \wedge \neg \Box 0 \\ \Box 1 \rightarrow \Box 0}}$

Concrete illustration: Compatibility with specificity [👉]

(26) $\mathrm{O}_{\mathrm{ExhSgDA}}(\Box_S(a \vee b \vee c))$
$= \Box_S(a \vee b \vee c) \wedge$
$(\Box_S a \rightarrow \Box_S b \vee \Box_S c) \wedge$
$(\Box_S b \rightarrow \Box_S a \vee \Box_S c) \wedge$
$(\Box_S c \rightarrow \Box_S a \vee \Box_S b)$
✓total ignorance, ✓neg. specificity

(27) $\mathrm{O}_{\mathrm{ExhSgDA}}(\Box_S(0 \vee 1 \vee 2))$
$= \Box_S(0 \vee 1 \vee 2) \wedge$
$(\Box_S 0 \rightarrow \Box_S 1 \vee \Box_S 2) \wedge$
$(\Box_S 1 \rightarrow \Box_S 0 \vee \Box_S 2) \wedge$
$(\Box_S 2 \rightarrow \Box_S 0 \vee \Box_S 1)$
✓total ignorance, ✓neg. specificity

(28) $\mathrm{O}_{\mathrm{ExhNonSgDA}}(\Box_S(a \vee b \vee c))$
$= \Box_S(a \vee b \vee c) \wedge$
$(\Box_S(a \vee b) \rightarrow \Box_S(a \vee c) \vee \Box_S(b \vee c)) \wedge$
$(\Box_S(a \vee c) \rightarrow \Box_S(a \vee b) \vee \Box_S(b \vee c)) \wedge$
$(\Box_S(b \vee c) \rightarrow \Box_S(a \vee b) \vee \Box_S(a \vee c))$
✓total ignorance, ✓pos. specificity

(29) $\mathrm{O}_{\mathrm{ExhNonSgDA}}(\Box_S(0 \vee 1 \vee 2))$
$= \Box_S(0 \vee 1 \vee 2) \wedge$
$(\Box_S(0 \vee 1) \rightarrow \Box_S(0 \vee 2) \vee \Box_S(1 \vee 2)) \wedge$
$(\Box_S(0 \vee 2) \rightarrow \Box_S(0 \vee 1) \vee \Box_S(1 \vee 2)) \wedge$
$(\Box_S(1 \vee 2) \rightarrow \Box_S(0 \vee 1) \vee \Box_S(0 \vee 2))$
✓total ignorance, ✓pos. specificity

Concrete illustration: Polarity sensitivity

(30) $\mathrm{O}_{\mathrm{ExhDA}}(\neg(a \vee b))$
$= \neg(a \vee b) \wedge$
$\neg \underbrace{(\neg a \wedge \neg\neg b)}_{\text{already excluded}} \wedge \neg \underbrace{(\neg b \wedge \neg\neg a)}_{\text{already excluded}}$

✗vacuous

(31) $\mathrm{O}_{\mathrm{ExhDA}}(\neg(0 \vee 1))$
$= \neg(0 \vee 1) \wedge$
$\neg \underbrace{(\neg 0 \wedge \neg\neg 1)}_{\text{already excluded}} \wedge \neg \underbrace{(\neg 1 \wedge \neg\neg 0)}_{\text{already excluded}}$

✗vacuous

Concrete illustration: Compatibility with some DE environments

(32) $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}} \forall w[(a \vee b)_w \rightarrow W_w]$
$= \forall w[(a \vee b)_w \rightarrow W_w] \wedge \exists w[(a \vee b)_w] \wedge$
$(\cdots \wedge \exists w[a_w]) \rightarrow (\cdots \wedge \exists w[b_w]) \wedge$
$(\cdots \wedge \exists w[b_w]) \rightarrow (\cdots \wedge \exists w[a_w])$
✓not vacuous

(33) $\mathrm{O}^{\mathrm{S}}_{\mathrm{ExhDA}} \forall w[(0 \vee 1)_w \rightarrow W_w]$
$= \forall w[(0 \vee 1)_w \rightarrow W_w] \wedge \exists w[(0 \vee 1)_w] \wedge$
$(\cdots \wedge \exists w[0_w]) \rightarrow (\cdots \wedge \exists w[1_w]) \wedge$
$(\cdots \wedge \exists w[1_w]) \rightarrow (\cdots \wedge \exists w[0_w])$
✓not vacuous

Concrete illustration: Scalar implicatures

(34) $O_{SA}\Box_S(a \vee b \vee c)$
$= \Box_S(a \vee b \vee c) \wedge \neg\Box_S(a \wedge b) \wedge \cdots \wedge \neg\Box_S(a \wedge b \wedge c)$

(35) $O_{SA}\Box_S(0 \vee 1 \vee 2)$
$\Box_S(0 \vee 1 \vee 2) \wedge \neg\Box_S(0 \vee 1)$

(36) $\Box_S O_{SA}(a \vee b \vee c)$
$= \Box_S((a \vee b \vee c) \wedge \neg(a \wedge b) \wedge \cdots \wedge \neg(a \wedge b \wedge c))$

(37) $\Box_S O_{SA}(0 \vee 1 \vee 2)$
$\Box_S((0 \vee 1 \vee 2) \wedge \neg(0 \vee 1))$
$= \Box_S 2$
clash with ignorance from O_{ExhDA}!

Q2: What is the parallel account?

It is an alternative-based account, though much more unified than in previous literature:

In all of indefinites, disjunction, and numerals we have obligatory:

- (except for BNs:) O_{ExhDA}:
  - ± ability to tolerate O_{ExhDA} relative to just natural subsets—SgDA, NonSgDA
  - ± ability to tolerate O_{ExhDA} that does not lead to a properly stronger meaning
- (including BNs) O_{SA}

Outline

Preamble

Q1: Are the facts parallel?

Q2: What is the parallel account?

Q3: Why should we care?

Montague:
Natural languages are logical!

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Others:
Natural languages are supralogical = idiosyncratic, illogical.

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- consequence: tempting to give up on supralogical = principled, very logical

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- impact: very successful for many effects in many categories of language
- challenge: supralogical effects rich, vary both between and within categories of language
- consequence: tempting to give up on supralogical = principled, very logical
- main point today: we don't have to

Thank you!

Appendix

(38) Jo called some student.

$\exists x \in [\![\text{student}]\!][C(j,x)]$ (assertion)

(39) Jo called a, b, …, or …

$\bigvee_{x \in \{a,b,\ \ldots\}} C(j,x) \Leftrightarrow C(j,a) \vee C(j,b) \vee \ \ldots$ (assertion)

(40) Three people quit.

$\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (assertion)

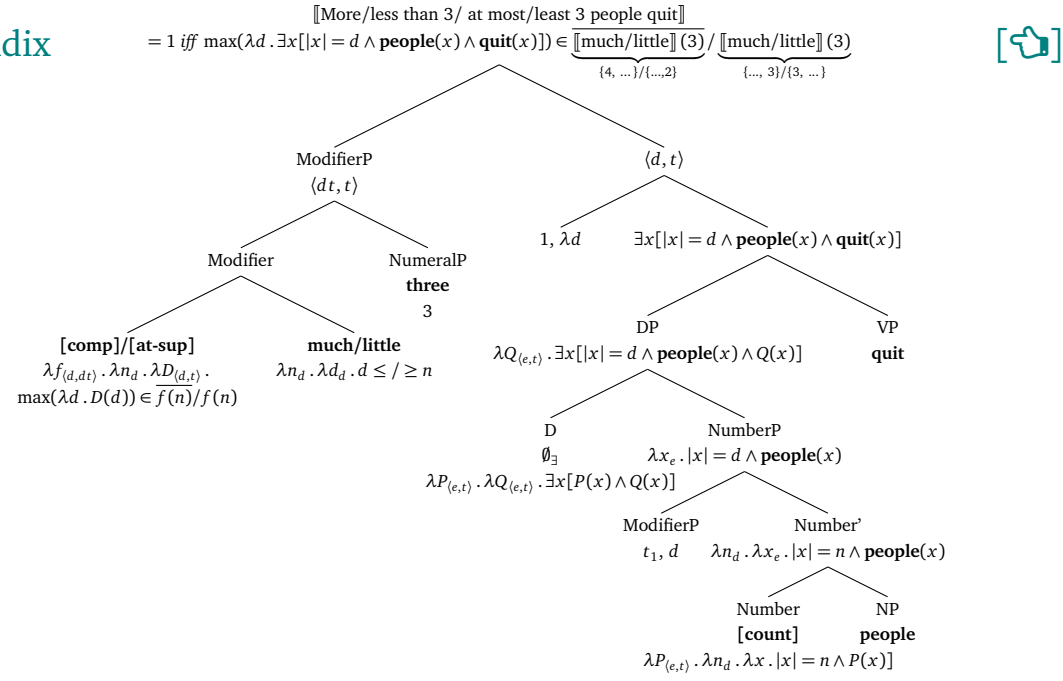
(41) More/less than 3 people quit.

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(3)}^{\{4,\ \ldots\}/\{\ldots,\ 2\}}$ (assertion)

(42) At most/least 3 people quit.

$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(3)}^{\{\ldots,\ 3\}/\{3,\ \ldots\}}$ (assertion)

Appendix


⟦More/less than 3/ at most/least 3 people quit⟧
= 1 iff max(λd . ∃x[|x| = d ∧ people(x) ∧ quit(x)]) ∈ ⟦much/little⟧ (3) / ⟦much/little⟧ (3)
{4, ...}/{...,2}
{..., 3}/{3, ...}
ModifierP
⟨dt, t⟩
⟨d, t⟩
Modifier
NumeralP
three
3
1, λd
∃x[|x| = d ∧ people(x) ∧ quit(x)]
[comp]/[at-sup]
λf⟨d,dt⟩ . λnd . λD⟨d,t⟩ .
max(λd . D(d)) ∈ f(n)/f(n)
much/little
λnd . λdd . d ≤ / ≥ n
DP
VP
λQ⟨e,t⟩ . ∃x[|x| = d ∧ people(x) ∧ Q(x)]
quit
D
NumberP
∅∃
λxe . |x| = d ∧ people(x)
λP⟨e,t⟩ . λQ⟨e,t⟩ . ∃x[P(x) ∧ Q(x)]
ModifierP
Number'
t1, d
λnd . λxe . |x| = n ∧ people(x)
Number
NP
[count]
people
λP⟨e,t⟩ . λnd . λx . |x| = n ∧ P(x)]

scenarios of interest

total variation	partial variation		no variation	
‘no winner’	neg. specificity ‘one loser’	pos. specificity ‘one winner’-1	pos. specificity ‘one winner’-2	‘all winners’
e.g.,	e.g.,	e.g.,	e.g.,	e.g.,
w_1: x ~~y~~ ~~z~~	w_1: ~~x~~ y ~~z~~	w_1: x y ~~z~~	w_1: x ~~y~~ ~~z~~	w_1: x y z
w_2: ~~x~~ y ~~z~~	w_2: ~~x~~ ~~y~~ z	w_2: x ~~y~~ z	w_2: x ~~y~~ ~~z~~	w_2: x y z
w_3: ~~x~~ ~~y~~ z	w_3: ~~x~~ y z	w_3: x y z	w_3: x ~~y~~ ~~z~~	w_3: x y z

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