Negative comparison between exactness, ignorance, and evaluativity

(Or how to be judgmental and ignorant with scalar alternatives)

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@ Penn Linguistics Conference (PLC) 45 | U of Pennsylvania (online) | Mar 19-21, 2021

Outline

Two negative comparisons, one judgmental and one ignorant[2,3]

(1) Jo found **no more than 10** marbles.
= She found $\neg > 10 = \leq 10$ marbles.
$\rightsquigarrow$ She found exactly 10. (EX)
$\rightsquigarrow$ Speaker thinks this is few. (EVAL)

(2) Jo found **not more than 10** marbles.
= She found $\neg > 10 = \leq 10$ marbles.
$\not\rightsquigarrow$ She found exactly 10. (NO EX)
$\rightsquigarrow$ Speaker not sure how many. (NEG-IG[1])

[1] Calling it NEG-IG to distinguish it from the ignorance effect of modified numerals in positive, seemingly episodic contexts, POS-IG, as I believe they might have different sources.

[2] Cf. observations in Nouwen (2008:277, 286) (who cites Jespersen 1949, 1966, who cites Stoffel 1894) and Mayr (2013:11-14). Nouwen mostly focuses on EX.

[3] Throughout we will speak of *more than n* (and *at least n*), but we will have in mind *less than n* (and *at most n*) also.

Outline

The puzzle

Three solutions: Horn (1972) and two variants of *Altered* Horn (1972)

The fourth solution: *Enhanced* Horn (1972)

Conclusion and outlook

S1: Nouwen (2008) using Horn (1972) (Nouwen actually rejects this)

- **Focus:** ***no more than n*: EX:**

  (1) Jo found **no more than 10** marbles.
  $=$ She found $\neg >$ 10 marbles.
  $\rightsquigarrow$ She found exactly 10.

- **Connection:** ***n*: EX:**

  (3) Jo found **10** marbles.
  $=$ She found $\geq$ 10 marbles.
  $\rightsquigarrow$ She found exactly 10.
  Horn (1972):
  Note that some items belong to natural scales. E.g., 2 belongs to $\langle 0, 1, 2, \ldots \rangle$.
  Assume uttering a scalar item makes salient its scalemates = its scalar alternatives.
  Assume Gricean reasoning negates any non-entailed alternative.

  Captures *n*: EX. **Captures *no more than n*: EX.**

- **Issue 1: Predicts *(not) more than n* / *at least n* systematically || to *n*, but: not so.** [ ]
  (Krifka 1999, Fox and Hackl 2006)

- **Issue 2: Predicts *not more than n* to have EX, but: NO EX.** (see starting puzzle)

S2: Nouwen (2008) using Fox and Hackl (2006)

- **Focus:** ***no more than n*: EX**

  (1) Jo found **no more than 10** marbles.
  $=$ She found $\neg >$ 10 marbles.
  $\rightsquigarrow$ She found exactly 10.

- **Connection:** ***more than n*: NO EX:**

  (4) Jo found **more than 10** marbles.
  $=$ She found $>$ 10 marbles.
  $\not\rightsquigarrow$ She found exactly 11.
  Fox and Hackl (2006): (alteration of Horn 1972 alt's)
  Assume numerals always belong to dense scales, that is, $\mathbb{R}$.
  This yields EX for non-strict comparison but $\perp$ for strict comparison.

  Captures *more than n*: NO EX. **Captures *no more than n*: EX.**

- **Issue 1: Predicts *more than n* systematically $\not\parallel$ from *at least n*, but: not so.** []
  (Mayr 2013:11-14)

- **Issue 2: Predicts *not more than n* to have EX, but: NO EX.** (see starting puzzle)

S3: Mayr (2013)

- **Focus: *not more than n*: NO EX:** (*more than n* & *at least n* more generally)

  (2) Jo found **not more than 10** marbles.
  $=$ She found $\neg >$ 10 marbles.
  $\not\rightsquigarrow$ She found exactly 10.

- **Connection: *more than n* / *at least n*: NO EX:** (*more than n* || *at least n* more generally)

  (5) Jo found **more than 10 / at least 10** marbles.
  $=$ She found $>$ / $\geq$ 10 marbles.
  $\not\rightsquigarrow$ She found exactly 11 / 10.
  Mayr (2013): (alteration of Horn 1972 alt's; also of which alt's are excluded)
  Assume modified numerals (MNs) get scalar alternatives based on replacing not just the numeral but also the modifier with another modifier of the same type.
  This yields NO EX for both *MN* and *not MN*.

  Captures *more than n* / *at least n*: NO EX. **Captures *not more than n*: NO EX.**

- **Issue 1: Predicts *more than n* / *at least n*: no 2nd bound, but: not so.** [☞]
  (Cummins et al. 2012)

- **Issue 2: Predicts *no more than n* to have NO EX, but: EX.** (see starting puzzle)

Looking back

	EX	NO EX	NEG-IG	EVAL	Issues:
S1	✓	✗	✗	✗	Predicts *(not) more than / at least n* systematically $\parallel$ to n, but: not so.
S2	✓	✗	✗	✗	Predicts *more than n* systematically $\nparallel$ from *at least n*, but: not so.
S3	✗	✓	✗	✗	Predicts *more than n / at least n*: no 2nd bound, but: not so.

Three solutions. Essentially: Horn (1972) and two variants of *Altered* Horn (1972)…

All have issues…

None can capture both EX and NO EX, or how negative comparison can go either way…

None has a solution for IG and EVAL…

Outline

The puzzle

Three solutions: Horn (1972) and two variants of *Altered* Horn (1972)

The fourth solution: *Enhanced* Horn (1972)

Conclusion and outlook

Looking forward

	EX	NO EX	NEG-IG	EVAL	Issues:
S1	✓	✗	✗	✗	Predicts *(not) more than / at least n* systematically $\parallel$ to *n*, but: not so.
S2	✓	✗	✗	✗	Predicts *more than n* systematically $\nparallel$ from *at least n*, but: not so.
S3	✗	✓	✗	✗	Predicts *more than / at least n*: no 2nd bound, but: not so.

- **Focus: $\neg > n$: EX, NO EX, NEG-IG, EVAL**

- **Connection 1: *more than n / at least n*: POS-IG:**

  (6) A: How many kids do you have? B: ??More than 3. / ??At least 3. (Mayr and Meyer 2014)
  Suggests *more than n / at least n*: NO EX due to a clash between EX and POS-IG. (Mihoc 2021)
  Suggests it is safe to assume *more than n / at least n* generally $\parallel$ *n*. (Mihoc 2021)
  Suggests it is safe to model *no more than n*: EX on *n*: EX, against S2.

- **Connection 2: *n*: EX, but *not n*: NO EX**

  (7) Peter didn't solve 10 problems. $\not\rightsquigarrow$ exactly 9 (Spector 2013:279-80)
  Suggests it is not safe to model *not more than n*: NO EX on *more than n*: NO EX, against S3.

Issues with S2-S3 seem to grow … Issues with S1 seem to diminish… *Enhanced* Horn (1972)?

Background assumptions

Truth conditions:

(8) $[\![\text{more than 3 P Q}]\!] = \underbrace{\max(\lambda d_d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)] \in \overbrace{[\![\text{much}]\!](3)}^{\{4,5,\dots\}}}_{\text{abbreviated: } 4\vee 5\vee\dots}$ (Mihoc 2020)

(9) $[\![\text{no / not more than n P Q}]\!] = \neg([\![\text{more than n P Q}]\!])$

Alternative generation:

- **Scale-related:** Replace the scalar element with its scalemates. Yields SA. (Horn 1972)
- **Domain-related:** Replace the domain with its subsets. Yields DA. (Chierchia 2013)

Alternative use:

(10) $[\![\mathrm{O}]\!](C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s) = p(w) \wedge \forall q \in C[q(w) \rightarrow p \subseteq q]$ (Chierchia 2013)

(11) $[\![\mathrm{E}]\!](C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s) = p(w) \wedge \underbrace{\forall q \in C[q \neq p \rightarrow p \prec_\mu q]}_{\text{usually assumed to be presupposed}}$ (Crnič 2012, Chierchia 2013)

Deriving EX (rehabilitation of S1)

In both positive and negative contexts: As in Horn (1972), updated with O:

(12) $[\![O_{SA}(\text{Jo found 10 marbles})]\!]$
$= (10 \vee \ldots) \wedge \neg(11 \vee \ldots)$
$= 10$ ✓

(13) $[\![O_{SA}(\text{Jo found more than 10 marbles})]\!]$
$= (11 \vee \ldots) \wedge \neg(11 \vee \ldots)$
$= 11$ ✗ (not a problem because blocked by POS-IG, derived from O_{DA} and $\Box_S$)

(14) $[\![O_{SA}(\text{Jo found no more than 10 marbles})]\!]$
$= \neg(11 \vee \ldots) \wedge \neg\neg(10 \vee \ldots)$
$= (10 \vee \ldots) \wedge \neg(11 \vee \ldots)$
$= 10$ EX ✓

Prediction: If there is no clash with O_{DA}, *more than n* can give rise to SA-implic's.

Deriving NO EX and NEG-IG

Additional assumption: Alternative generation: [☞]

- **Scale- & *structure*-related, SA+:** Replace scalar. Delete negation. (∼Fox and Katzir 2011)

(15) $[\![\mathrm{O}_{SA+}(\text{Jo found not more than 2 marbles})]\!]$

$$= \neg(3 \vee 4 \vee \ldots) \wedge \underbrace{\neg\neg(2 \vee \ldots) \wedge \neg\neg(1 \vee \ldots) \wedge \ldots}_{\text{implic's from scale-related alt's}} \wedge \underbrace{\neg(2 \vee \ldots) \wedge \neg(1 \vee \ldots) \wedge \ldots}_{\text{implic's from structure-related alt's}} = \bot \qquad ✗$$

Additional assumption: Alternative use: (also used for POS-IG)

- **Rescue mechanism:** If O yields ⊥, try inserting $\Box_S$. (Kratzer and Shimoyama 2002)

(16) $[\![\mathrm{O}_{SA+}(\text{Jo found not more than 2 marbles})]\!]$

$$= \neg(3 \vee 4 \vee \ldots) \wedge \underbrace{\neg\Box_S\neg(2 \vee \ldots) \wedge \neg\Box_S\neg(1 \vee \ldots) \wedge \ldots}_{\text{implic's from scale-related alt's}} \wedge \underbrace{\neg\Box_S(2 \vee \ldots) \wedge \neg\Box_S(1 \vee \ldots) \wedge \ldots}_{\text{implic's from structure-related alt's}} \quad \Rightarrow \text{NEG-IG} ✓$$

Additional assumption: *No more than n* cannot delete its negation.

Prediction: We should still see effects from this mechanism in other items. [☞]

Deriving EVAL

As suggested in Crnič (2011) and references therein, comes from E(ven):

Additional assumptions: Alternative use: E [☞]

- If O(nly) targets the non-entailed SA, E(ven) targets the *entailed* SA. (Mihoc 2020)
- E(ven) uses the prejacent and SA pre-exhaustified. (Mihoc 2021, cf. Crnič 2012 for *even*)

(17) $[\![\mathrm{E}_{\mathrm{SA}}(\text{Jo found no more than 10 marbles})]\!]$
$= \underbrace{\mathrm{O}_{\mathrm{SA}}(\neg(10 \vee \ldots))}_{\text{'exactly 10'}} \wedge \underbrace{\mathrm{O}_{\mathrm{SA}}(\neg(10 \vee \ldots)) \prec_{\mu} \mathrm{O}_{\mathrm{SA}}(\neg(11 \vee \ldots))}_{\text{'exactly 10 is less likely than exactly 11': 'This is few!'}}$ EX, EVAL ✓

Additional assumption: *Not more than n* does not undergo exhaustification via E(ven).

Prediction: We should still see effects from this mechanism in other items. [☞]

Outline

The puzzle

Three solutions: Horn (1972) and two variants of *Altered* Horn (1972)

The fourth solution: *Enhanced* Horn (1972)

Conclusion and outlook

Conclusion

no more than n and *not more than n*:

- look very similar
- are generally assume to have the same semantics
- exhibit very different effects: EX and EVAL vs. NO EX and NEG-IG

Existing solutions:

- narrow focus
- non-conservative
- non-integrated
- do not capture all the patterns

Today's solution:

- broader focus
- conservative
- integrated
- captures all the patterns

Outlook

Re. the puzzle:

- *No/not Adj-er than x* seems to have similar patterns. This is predicted.
- Why does *no more than n* not allow negation deletion?
- Cross-linguistically, can we find more evidence of pairs of expressions that differ like this?
- Why is this contrast based on comparative structures? (SMNs tend to be PPIs; why?)

Re. the solution for EX:

- Any other necessary refinements of Horn (1972)?

  For example, scale truncations, between-item variation, etc.

Re. the solution to NO EX—IG/NEG-IG:

- Any other necessary qualifications of the structural theory?

  We argued for a 3-way distinction between scalar, subdomain, and structural.

Re. the solution to EVAL:

- Any other necessary refinements of E(ven)?

Thank you!

Appendix: The puzzle

(Nouwen 2008:277)

According to the semantics above, *no -er* constructions express $\leq$- or $\geq$-relations. In reality, however, such constructions behave as if they were comparatives expressing *equality*.[4] This already becomes clear from such simple examples as (21). Its preferred interpretation is that Cody found exactly ten marbles. Moreover, it has an evaluative side-effect of expressing that ten marbles does not count as a lot. Similarly, (22) says that fifty people showed up and that this can be considered to be many.

(22) (The organisers expected a small audience. However,) no fewer than fifty people showed up.

I am not the first person to notice such data.[5] Jespersen (1966), for instance, remarks that "*no less than 30* means exactly 30, implying surprise or wonder at the high number" (p. 83). Elsewhere (Jespersen 1949, entry 16.842 on p. 434), he equates *no more than* to *as little as* and explores the full range of uses of the *no more* construction (entries 16.83–16.86). Jespersen notes that there is a difference between *no* and *not* in combinations with comparatives. This contrast, he notes, had already been observed by Stoffel (1894), who for instance discussed the quote "The victorious emperor remained at Rome not more than three months." Stoffel comments on it in the following way: "This means that he remained three months *at most*; if the author had written 'no more than three months', this form of expression would have implied that the author thought this a brief period, and 'no more than three months' would be equivalent to 'three months *only*'" (Jespersen 1949, p. 435).

Appendix: *More than / at least n* $\nparallel$ *n*

(Krifka 1999, Fox and Hackl 2006, concise summary from Mayr 2013)

(Krifka 1999:258) notes that sentences embedding *at least n* – that is, numerals modified by the superlative *at least* – do not lead to an implicature that *not at least n+1*. If (7) licensed such an inference, it would be taken to imply that Jack read exactly three books.

(7) Jack read at least three books.
$\not\leadsto$ Jack did not read at least four books

(Fox and Hackl 2006:540) note that numerals n modified by comparative *more than* similarly do not give rise to an implicature that *not more than n+1*. Otherwise (8) would imply that Jack read exactly four books.

(8) Jack read more than three books.
$\not\leadsto$ Jack did not read more than four books

Appendix: *More than n* || *at least n*

(Mayr 2013)

(Fox and Hackl 2006:544) note that universal modals reintroduce the implicature of *more than n*, (14), whereas existential modals do not do so, (15). In other words, in the former situation an exact interpretation of the numeral seems to become available.

(14) Jack is required to read more than three books.
$\leadsto$ There is no degree d larger than 3 such that Jack is required to read more than d-many books

But Fox and Hackl (2006) do not note that *at least n* shows a behavior parallel to the one of *more than n*. Only in a sentence where *at least n* is embedded under a universal modal does the exact interpretation become possible:

(18) Jack is required to read at least three books.
$\leadsto$ There is no degree d greater than 3 such that Jack is required to read at least d-many books

3.3 Embedding under negation

Let us now consider modified numerals embedded under negation, having in mind the surface scope interpretation of the sentences discussed. First note that negative environments like other DE-contexts change the entailment patterns. Therefore the stronger alternatives to, say, 3 are now all the degrees smaller than 3. Consider (22a) under this aspect first. It does not seem to imply that Jack read exactly three books, which would be expected if the implicature noted were available.[9] Notice moreover that *not more than n* is equivalent to *at most n*, where *at most* is DE. In other words, (22a) is equivalent to (22b), and it is clear that (22b) does not have the exact-interpretation either.

(22) a. Jack didn't read more than three books.
$\not\leadsto$ Jack read more than two books
b. Jack read at most three books.
$\not\leadsto$ Jack didn't read at most two books

At present it is not clear to me how Fox and Hackl's 2006 approach could be modified in order to account for the parallelism between numerals modified by superlatives and those modified by comparatives. I therefore conclude that a new approach to the varying absence and presence of scalar implicatures for modified numerals is called for. In the following section, I will try to further strengthen the similarity between *at least n* and *more than n*.

Appendix: *More than n / at least n*: episodic contexts: SA-implic's

(Cummins et al. 2012)

Granularity and scalar implicature in numerical expressions

Chris Cummins · Uli Sauerland · Stephanie Solt

Published online: 18 July 2012

Abstract It has been generally assumed that certain categories of numerical expressions, such as 'more than *n*', 'at least *n*', and 'fewer than *n*', systematically fail to give rise to scalar implicatures in unembedded declarative contexts. Various proposals have been developed to explain this perceived absence. In this paper, we consider the relevance of scale granularity to scalar implicature, and make two novel predictions: first, that scalar implicatures are in fact available from these numerical expressions at the appropriate granularity level, and second, that these implicatures are attenuated if the numeral has been previously mentioned or is otherwise salient in the context. We present novel experimental data in support of both of these predictions, and discuss the implications of this for recent accounts of numerical quantifier usage.

These patterns can be interpreted as arising from scalar implicatures based on granularity. In completing the task, respondents appear to have relied on the inference that a decision not to refer to a higher value on a scale of the same or a coarser level of granularity is an indication that (as far as the speaker/writer knows) the statement with that higher value does not hold. That is, when presented with 'more than *n*', the hearer typically computes the implicature that 'not more than *m*', where *m* is the next higher point on a scale on which *n* occurs (or on a scale of coarser granularity). The coarser-grained the scale, the more 'distant' *m* is from *n*, and thus the higher respondents' estimates relative to *n*.

Appendix: Truth conditions

(Mihoc 2019, 2020, 2021)

(8) Three people quit.

a. $\exists x[|x| = 3 \wedge P(x) \wedge Q(x)]$ (assertion)

b. $\{\exists x[|x| = m \wedge P(x) \wedge Q(x)] \mid m \in S\}$ (SA)

(9) More/less than 3 people quit.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(3)}^{\{4,\,\ldots\}/\{\ldots,\,2\}}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ (SA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subsetneq [\![\text{much/little}]\!]\,(3)\}$ (DA)

(10) At most/least 3 people quit.

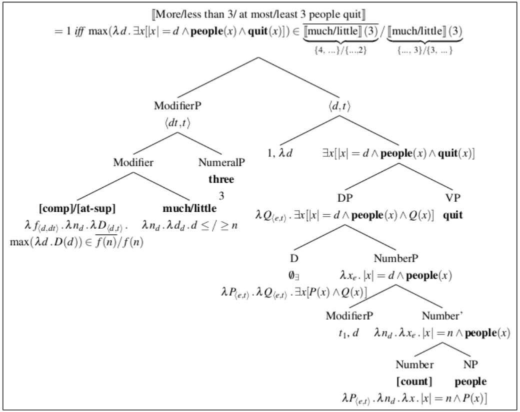
a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in \overbrace{[\![\text{much/little}]\!]\,(3)}^{\{\ldots,\,3\}/\{3,\,\ldots\}}$ (assertion)

b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ (SA)

c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge Q(x)]) \in D' \mid D' \subsetneq [\![\text{much/little}]\!]\,(3)\}$ (DA)

More than 2 / at least 3 people quit.		Less than 3 / at most 2 people quit.	
3 4 5 ...	(SgDA)	0 1 2	(SgDA)
$3\vee 4$ ... $5\vee 7\vee 11$ ...	(NonSgDA)	$0\vee 1$ $0\vee 2$ $1\vee 2$	(NonSgDA)
↓		↓	
$2\vee\ldots \leftarrow \mathbf{3\vee 4}\vee\ldots \leftarrow 4\vee\ldots \leftarrow \ldots$	(SA)	$0 \rightarrow 0\vee 1 \rightarrow \mathbf{0\vee 1\vee 2} \rightarrow \cdots\vee 3 \rightarrow \ldots$	(SA)

(b) The truth conditions (bottom center, black, bold) and alternatives (SA bottom left and right, blue; DA top, red) of BNs, CMNs, and SMNs, schematically. Those for MNs are shown explicitly, and those for BNs are the same as for the UE SMNs (e.g., *3* like *at least 3*), except BNs only get the SA. Arrows indicate direction of entailment.



(a) The syntax and semantics of BNs, CMNs, and SMNs. Those for MNs are shown explicitly, and those for BNs can be obtained by replacing ModifierP near Number' with NumeralP (from higher up). I assume a numeral, NumeralP, denotes a simple degree. In BNs, it combines with Number' to yield a predicative meaning, which then undergoes existential closure; this is the standard existential, 'at least' meaning of BNs cf. Link (1987) (and other literature since). In CMNs/SMNs, it combines with [comp]/[at-sup] and *much/little* (analyzed as positive/negative extent indicators, adapting ideas from Seuren 1984 or Kennedy 1997, 2001), yielding a generalized quantifier over degrees, which cannot combine with Number' but has to move, leaving behind a trace type d; this is the meaning of CMNs from Heim (2000)-Hackl (2000) and of CMNs and SMNs from Kennedy (2015), though the assumptions about [comp]/[at-sup] and *much/little* are new. For assumptions about NumberP and NumeralP see Zabbal (2005) or Scontras (2013), and references therein; ModifierP is new.

Appendix: *More than n / at least n*: NO EX: blocked by IG

(Mihoc 2019, 2020, 2021)

(77) John called less than three people / at most two people.

$\mathrm{O}_{\mathrm{ExhDA}}(\Box_S \mathrm{O}_{\sigma A}(0 \vee 1 \vee 2))$

a. $\Box_S \mathrm{O}_{\sigma A}(0 \vee 1 \vee 2) \wedge$

b. $\neg \mathrm{O}\Box_S 0 \wedge \neg \mathrm{O}\Box_S 1 \wedge \neg \mathrm{O}\Box_S 2 \wedge \neg \mathrm{O}\Box_S (0 \vee 1) \wedge \neg \mathrm{O}\Box_S (1 \vee 2) \wedge \neg \mathrm{O}\Box_S (0 \vee 2)$

$$= \underbrace{\underbrace{\overbrace{\Box_S((0\vee 1\vee 2)\wedge\neg(0\vee 1))}^{(a)}}_{=\Box_S 2} \wedge \overbrace{\neg\Box_S 0\wedge\neg\Box_S 1\wedge\neg\Box_S 2}^{(b)}}_{\perp}$$

As we can see, the assertion yields certainty about the domain, $\Box_S(0 \vee 1 \vee 2)$. Then $\mathrm{O}_{\sigma A}$ strengthens it to certainty about, essentially, a singleton DA, $\Box_S 2$. But $\mathrm{O}_{\mathrm{ExhDA}}$ yields total ignorance (the default result in default contexts), including ignorance about this singleton DA, $\neg\Box_S 2$. The result is a contradiction. This means that the illicit 'exactly' σA-implicature is in fact not generated, solving (61).

Appendix: EX: predictions

More than n and *at least n* can give rise to SA-implic's even in episodic contexts—but only with coarser scale granularity. (Cummins et al. 2012, Spector 2014)

This suggests that the clash between SA- and DA-implics triggers a repair mechanism.

I believe this mechanism is contextual scale truncation.

Appendix: NO EX—IG: predictions

Not n, *not Jo*, do seem to give rise to the same NO EX—IG patterns also:

(18) A: How many marbles did Jo find? B: Not 10—everyone who found 10 moved on to the next phase.

(19) A: Who did Mary collaborate with? B: Not Jo—she didn't do that project.

In both cases, classic SA-implic's predict a different meaning (exactly 9, everyone else), but what we get is NEG-IG.

Appendix: EVAL: predictions

(Mihoc 2021)

First, SMNs in plain UE contexts. Here, as discussed, SMNs give rise to IG, via $\mathrm{O}_{\mathrm{ExhDA}}$, and to SI, via $\mathrm{O}_{\mathrm{ExhDA}}$. However, they also give rise to an evaluative judgment—*at least 3*/*at most 3* suggests that 3 is many/few. Note that this can be derived from E_{SA}.

(31) Jo solved at least 3 problems.
exactly 3 $\prec_\mu$ exactly 2 $\rightsquigarrow$ 3 is many!

(32) Jo solved at most 3 problems.
exactly 3 $\prec_\mu$ exactly 4 $\rightsquigarrow$ 3 is few!

(33) Jo may drink ✓**at least** 3 beers.
$$\underbrace{\mathrm{E}_{\mathrm{ExhSA}}(\underbrace{\mathrm{O}_{\mathrm{SA}}(\underbrace{\mathrm{O}_{\mathrm{ExhDA}}(\Diamond(3 \vee 4 \vee \ldots))}_{(\Diamond 3 \wedge \Diamond 4 \wedge \ldots)\ \text{Free Choice}})}_{\neg(\Diamond 2 \wedge \Diamond 3 \wedge \ldots)\ \text{not possible less}})}_{\Diamond 3 \prec_\mu \Diamond 4\ \ \text{does not fit common assumptions}}$$

(34) Jo may drink ✓**at most** 3 beers.
$$\underbrace{\mathrm{E}_{\mathrm{ExhSA}}(\underbrace{\mathrm{O}_{\mathrm{SA}}(\underbrace{\mathrm{O}_{\mathrm{ExhDA}}(\Diamond(\cdots \vee 2 \vee 3))}_{(\cdots \wedge \Diamond 2 \wedge \Diamond 3)\ \text{Free Choice}})}_{\neg(\cdots \wedge \Diamond 3 \wedge \Diamond 4)\ \text{not possible more}})}_{\Diamond 3 \prec_\mu \Diamond 2\ \ \text{fits common assumptions}}$$

(29) a. If Jo solved ✓**at least 3** problems, she passed.
$$\underbrace{\mathrm{E}_{\mathrm{ExhSA}}(\text{If } \underbrace{\mathrm{O}_{\mathrm{SA}}(\text{Jo solved ✓at least 3 problems})}_{\text{exactly 3 solutions}}, \text{ she passed.}}_{\text{exactly 3 solutions} \to \text{pass} \prec_\mu \text{exactly 4/5/...solutions} \to \text{pass}}$$
b. If Jo solved # **at least 3** problems, she failed.
exactly 3 solutions $\to$ fail $\prec_c$ exactly 4 solutions $\to$ fail
c. If Jo didn't solve # **at least 3** problems, she passed.
exactly 3 solutions $\to$ pass $\prec_c$ exactly 2 solutions $\to$ pass
d. If Jo didn't solve ✓**at least 3** problems, she failed.
✓exactly 3 solutions $\to$ fail $\prec_c$ exactly 2 solutions $\to$ fail
e. If Jo made # **at least 3** mistakes, she passed.
exactly 3 mistakes $\to$ pass $\prec_c$ exactly 4 mistakes $\to$ pass
f. If Jo made ? **at least 3** mistakes, she failed.
✓exactly 3 mistakes $\to$ fail $\prec_c$ exactly 4 mistakes $\to$ fail
g. If Jo didn't make ? **at least 3** mistakes, she passed.
✓exactly 3 mistakes $\to$ pass $\prec_c$ exactly 2 mistakes $\to$ pass
h. If Jo didn't make # **at least 3** mistakes, she failed.
exactly 3 mistakes $\to$ fail $\prec_c$ exactly 2 mistakes $\to$ fail

(30) a. If Jo solved # **at most 3** problems, she passed.
exactly 3 solutions $\to$ pass $\prec_c$ exactly 2 solutions $\to$ pass
b. If Jo solved ✓**at most 3** problems, she failed.
✓exactly 3 solutions $\to$ fail $\prec_c$ exactly 2 solutions $\to$ fail
c. If Jo didn't solve ? **at most 3** problems, she passed.
✓exactly 3 solutions $\to$ pass $\prec_c$ exactly 4 solutions $\to$ pass
d. If Jo didn't solve # **at most 3** problems, she failed.
exactly 3 solutions $\to$ fail $\prec_c$ exactly 4 solutions $\to$ fail
e. If Jo made ✓**at most 3** mistakes, she passed.
✓exactly 3 mistakes $\to$ pass $\prec_c$ exactly 2 mistakes $\to$ pass
f. If Jo made # **at most 3** mistakes, she failed.
exactly 3 mistakes $\to$ fail $\prec_c$ exactly 2 mistakes $\to$ fail
g. If Jo didn't make # **at most 3** mistakes, she passed.
exactly 3 mistakes $\to$ pass $\prec_c$ exactly 4 mistakes $\to$ pass
h. If Jo didn't make ? **at most 3** mistakes, she failed.
✓exactly 3 mistakes $\to$ fail $\prec_c$ exactly 4 mistakes $\to$ fail

Appendix: Structure-related alternatives

(Katzir 2007, Fox and Katzir 2011:97)

As in the theories of Horn and Rooth, we will define the alternatives in terms of replaceable elements and their possible replacements. In order to account for focus sensitivity both in SI and in AF, we will follow Rooth in identifying the replaceable elements with the set of focused constituents. The possible replacements, however, will have to be able to break symmetry, which Rooth's alternatives cannot. Instead, we will follow the proposal in Katzir (2007)—originally stated for SI only but extended here also to AF—in identifying the possible replacements of a constituent with the set of all constituents that are at most equally complex, under a particular definition of complexity.

The definition of F in Katzir (2007) makes use of a notion of structural complexity in a given context. Simplifying somewhat, we can define a relation between structures $\precsim_C$, 'at most as complex as, in context C', as follows:

(34) $S' \precsim_C S$ if S' can be derived from S by successive replacements of sub-constituents of S with elements of the substitution source for S in C, $SS(S, C)$

(35) $SS(X, C)$, the substitution source for X in context C, is the union of the following sets:

a. The lexicon

b. The sub-constituents of X

c. The set of salient constituents in C [16]

Note1: Predicts an alternative to *more than n* could be *n*. I reject this.

Note 2: Predicts an alternative to *not more than n* could be *more than n*. I adopt this.

Appendix: E(ven)

(Mihoc 2021)

4.2. Extension: O–*only*, E–*even*, and O/*only*–E/*even*

Horn (1972: 37ff.) notes contrasts between what we now call instances of covert O and overt *only*—O is fine with *more* but bad with *less*, while *only* is the opposite. Building on comments from an anonymous reviewer, I similarly note contrasts between what we now call instances of covert E and overt *even*—E with *at least [positive P1]* is fine with a positive P2 but bad with a negative P2, which we said is because E pitches the prejacent up against its entailed SA, but *even* is fine with either a positive or a negative P2, which by our reasoning must come because *even* can pitch the prejacent up against either its entailed or its non-entailed SA.

(35) {O / Only} 60% of the electorate will be fooled, if not ✓/# more / #/✓less.

(36) {E / Even} if you solve at least 3 problems, you ✓/✓pass / #/✓fail.

Building on suggestions from Horn (1972) and literature since, I suggest that *only*/O are as follows: (1) They both convey the same set of inferences. However: (2) They differ in the status of some of these inferences (whether it is asserted, α, or presupposed, π). Moreover: (3) Their overt/covert status likely matters also. Analogously, and very tentatively, I suggest that *even*/E are as follows: (1) They both convey the same set of inferences. Above I said that, in light of non-end-of-the-scale items, the scalar inference cannot be about *all* the SA, and I suggested it is about the *entailed* SA. Here I suggest, in light of *even*, that it is restricted to the *true* SA, where a third commonly assumed existential inference ensures that at least one true alternative (different from the prejacent) is always available. Then: (2) They might also differ in the status of some of these inferences. With the literature, for now I will assume that they do not. However, crucially: (3) Their overt/covert status matters, as follows: With O_{SA} at the same site as E_{SA}, the set of true alternatives are the *entailed* SA (see (11), §3.1), which constrains the scalar inference such that sometimes it ends up clashing with context. Without O_{SA} at the same site as E_{SA}, the set of true alternatives can in principle be either the entailed or the non-entailed O_{SA}, which ensures that the scalar inference can adjust to context. Now, I assume that overt *even* can suspend O_{SA} at the same site, thus accessing the non-entailed SA, but covert E cannot. This affects their set of true alternatives, thus capturing their patterns.

(37) $[\![\text{only/O}]\!](C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s)$
  a. $p(w)$ (π/α)
  b. $\forall q \in C[q(w) \rightarrow p \subseteq q]$ (α/α)

(38) $[\![\text{even/E}]\!](C_{\langle\langle s,t\rangle,t\rangle}, p_{\langle s,t\rangle}, w_s)$
  a. $p(w)$ (α/α)
  b. $\forall q \in C[\underline{q(w)} \rightarrow p \prec_{\mu} q]$ (π/π)
  c. $\exists q \in C[\underline{q \neq p \wedge q(w)}]$ (π/π)

If this is on the right track, we can also draw conclusions about the role of the exhaustivity operators O-*only* and E-*even* more generally. When an assertion has multiple alternatives that are true, as is the case with the entailed SA of scalar items or with the additional true SA added by E-*even*, these operators clarify why the assertion was chosen over them: It entails any other true alternative (O-*only*) and/or it is more noteworthy than any other true alternative (E-*even*).

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